NUMERICAL ANALYSIS-EXERCISES
AA 2007-2008

Professor Eleuterio. F. Toro
Laboratory of Applied Mathematics
Department of Civil and Environmental Engineering
University of Trento
Trento, Italy
E-mail: toro@ing.unitn.it
Website: http://www.ing.unitn.it/toro

1 Model linear system with source terms

$$\partial_t \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} + \begin{bmatrix} 0 & -a \\ -a & 0 \end{bmatrix} \partial_x \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2}[(\beta_1+\beta_2)q_1 + (\beta_1-\beta_2)q_2 \\ \frac{1}{2}[(\beta_1-\beta_2)q_1 + (\beta_1+\beta_2)q_2 \end{bmatrix} , \tag{1}$$

where a is a positive constant, β_1, β_2 are negative constants.

Consider the with initial conditions

$$q_1(x,0) = q_1^{(0)}(x) \ , q_2(x,0) = q_2^{(0)}(x) \tag{2}$$

where $q_1^{(0)}(x)$ and $q_2^{(0)}(x)$ are two prescribed functions of x. The exact solution at a general point (x,t) is

$$\left. \begin{array}{l} q_1(x,t) = \frac{1}{2}[q_1^{(0)}(x-\lambda_1 t) + q_2^{(0)}(x-\lambda_1 t)]e^{\beta_1 t} + \frac{1}{2}[q_1^{(0)}(x-\lambda_2 t) - q_2^{(0)}(x-\lambda_2 t)]e^{\beta_2 t} \ , \\ q_2(x,t) = \frac{1}{2}[q_1^{(0)}(x-\lambda_1 t) + q_2^{(0)}(x-\lambda_1 t)]e^{\beta_1 t} - \frac{1}{2}[q_1^{(0)}(x-\lambda_2 t) - q_2^{(0)}(x-\lambda_2 t)]e^{\beta_2 t} \ . \end{array} \right\} \tag{3}$$

TASKS:

- Verify that (3) is the exact solution of the IVP (1)-(2) for $\lambda_1 = -a$, $\lambda_2 = a$ and for any constants β_1 and β_2.
- Choose the parameters as follows $a = 1$, $\beta_1 = \beta_2 = -1$, the computational domain as $[-5, 5]$ and the initial conditions as

  $$\left. q_1(x,0) = q_1^{(0)}(x) = e^{-x^2} \ , \ q_2(x,0) = q_2^{(0)}(x) = -\tfrac{1}{2}e^{-x^2} \ . \right\} \tag{4}$$

  Apply the ADER2 method with ENO reconstruction, component by component, to solve the above problem numerically using a mesh of 100 cells and a CFL number $C_{cfl} = 0.9$. Evolve the solution up to the output time $t_{out} = 2$ and compare the numerical with the exact solution.
- Discuss boundary conditions.
- Vary the CFL number, keeping the remaining parameters unchanged, and observe the effect on the numerical solution.
- Vary the mesh (eg. 200, 400, 800), keeping the remaining parameters unchanged, and observe the effect on the numerical solution.
- Set all slopes to zero in the programm so as to obtain Godunov's method and perform all the computational tasks given above. Discuss your observations.