Computational Methods for Mechatronics [140466] — 2015, January 16

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1

Compute the control for the following optimal control problem

$$
\begin{cases}
\text{Minimize:} \quad \displaystyle\int_0^1 x(t)^2 + y(t)^2 \,\mathrm{d}t \\
\text{Integral constraint:} \quad \displaystyle\int_0^1 x(t)\,\mathrm{d}t = 3 \\
x'(t) = x(t) + y(t) \\
y'(t) = u(t) \\
x(0) = 0, \quad x(1) = 0, \quad -1 \leq u(t) \leq 1
\end{cases}
$$

The Boundary Value Problem (BVP):
The optimal control u as a function of states and multiplier (Pontryagin):

2

Compute the control for the following free time optimal control problem

$$\begin{cases} \text{Minimize:} \quad \displaystyle\int_0^T u(t)^2 \,\mathrm{d}t \\ x'(t) = y(t) - x(t) + u(t) \\ y'(t) = x(t) - u(t) \\ x(0) = 0, \quad x(T) = 1, \quad y(0) = 0 \end{cases}$$

Suggention: First make a change of variable to eliminate the free time T and transform to problem to a fixed boundary problem.

The transformed (OCP):
The Boundary Value Problem (BVP):
The optimal control u as a function of states and multiplier (Pontryagin):

3

Study the following constrained minimization problem.

$$\text{minimize} \quad f(x,y,z) = x^2 + z^2 + z + x, \qquad z - y + 1 = 0, \qquad z \geq 2,$$

KKT system of first order condition:

$$\begin{cases} 2x+1 &= 0 \\ \lambda &= 0 \\ -\mu - \lambda + 2z + 1 &= 0 \\ z - y + 1 &= 0 \\ \mu(z-2) &= 0 \\ \mu &\geq 0 \end{cases}$$

Solutions of KKT system:

$$\begin{cases} x = -1/2,\ y = 3,\ z = 2,\ \lambda = 0,\ \mu = 5 \\ x = -1/2,\ y = 1/2,\ z = -1/2,\ \lambda = 0,\ \mu = 0 \end{cases}$$

Discussion of the stationary point: $x = -1/2,\ y = 3,\ z = 2,\ \lambda = 0,\ \mu = 5.$

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Solve the following recurrence

$$x_{k+1} = y_k - k, \qquad x_0 = 1$$
$$y_{k+1} = x_k + k, \qquad y_0 = 1$$

$\mathcal{Z}-$transform:

$$\zeta x(\zeta) - \zeta = y(\zeta) - \zeta/(\zeta-1)^2$$
$$\zeta y(\zeta) - \zeta = x(\zeta) + \zeta/(\zeta-1)^2$$

Solution in $\mathcal{Z}$:

$$x(\zeta) = \frac{\zeta(\zeta^2-2)}{(\zeta+1)(\zeta-1)^2} = -\frac{\zeta}{2(\zeta-1)^2} - \frac{\zeta}{4(\zeta+1)} + \frac{5\zeta}{4(\zeta-1)},$$
$$y(\zeta) = \frac{\zeta^3}{(\zeta+1)(\zeta-1)^2} = \frac{\zeta}{2(\zeta-1)^2} + \frac{\zeta}{4(\zeta+1)} + \frac{3\zeta}{4(\zeta-1)},$$

Solution in k

$$x_k = \frac{5}{4} - \frac{1}{2}k - \frac{1}{4}(-1)^k,$$
$$y_k = \frac{3}{4} + \frac{1}{2}k + \frac{1}{4}(-1)^k,$$

5

Given the following system of ODE:

$$\begin{cases} x''(t) - y(t) = -1 \\ \\ y'(t) - x(t) = -t \\ \\ x(0) = 0, \qquad y(0) = 1, \qquad x'(0) = A, \\ \end{cases}$$

Compute constant A in such a way $x(1) = 1$.

Laplace transform:

$$\begin{cases} s^2 x(s) - A - y(s) &= -\dfrac{1}{s} \\ sy(s) - 1 - x(s) &= -\dfrac{1}{s^2} \end{cases}$$

Solution in s:

$$\begin{cases} x(s) &= \dfrac{As^3 - 1}{s^2(s^3-1)} = \dfrac{1}{3}\dfrac{A-1}{s-1} + \dfrac{1}{3}\dfrac{s(1-A)+A-1}{s^2+s+1} + \dfrac{1}{s^2}, \\ y(s) &= \dfrac{s^3 + (A-1)s - 1}{s(s^3-1)} = \dfrac{1}{3}\dfrac{A-1}{s-1} + \dfrac{1}{s} + \dfrac{1}{3}\dfrac{(1-A)s - 2*A+2}{s^2+s+1}, \end{cases}$$

Solution in t:

$$\begin{cases} x(t) &= t + \dfrac{A-1}{3}\left(e^t + \left(\sqrt{3}\sin\dfrac{\sqrt{3}t}{2} - \cos\dfrac{\sqrt{3}t}{2}\right)e^{-t/2}\right) = t, \\ y(t) &= 1 + \dfrac{A-1}{3}\left(e^t - \left(\sqrt{3}\sin\dfrac{\sqrt{3}t}{2} + \cos\dfrac{\sqrt{3}t}{2}\right)e^{-t/2}\right) = 1, \end{cases}$$

Constant A

$$A = 1$$