Computational Methods for Mechatronics [140466] — 2015, February 13

Surname | | Name | | Mat. number | |

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1

Compute the control for the following optimal control problem

$$
\begin{cases}
\text{Minimize:} \quad \displaystyle\int_0^1 x(t)^2 + u(t)^2 \,\mathrm{d}t \\
\text{Integral constraint:} \quad \displaystyle\int_0^1 t + x(t) \,\mathrm{d}t = 0 \\
x'(t) = x(t) + y(t) \\
y'(t) = u(t) \\
x(0) = 0, \quad x(1) = 0, \quad -1 \leq u(t) \leq 1
\end{cases}
$$

The Boundary Value Problem (BVP):

The optimal control u as a function of states and multiplier (Pontryagin):

2

Compute the control for the following free time optimal control problem

$$\begin{cases} \text{Minimize:} \quad \displaystyle\int_0^T u(t)^2 + x(t)^2 \,\mathrm{d}t \\ x'(t) = y(t) + u(t) \\ y'(t) = x(t) - u(t) \\ x(0) = 0, \quad x(T) = 1, \quad y(0) = 0 \end{cases}$$

Suggention: First make a change of variable to eliminate the free time T and transform to problem to a fixed boundary problem.

The transformed OCP with fixed boundary:
The Boundary Value Problem (BVP):
The optimal control u as a function of states and multiplier (Pontryagin):

3

Study the following constrained minimization problem.

$$\text{minimize}\quad f(x,y,z) = x^2 + x + yz, \qquad z - y + 1 = 0, \qquad z \geq 2,$$

KKT system of first order condition:

$$\begin{cases} 2x+1 &= 0 \\ z+\lambda &= 0 \\ y-\lambda-\mu &= 0 \\ z-y+1 &= 0 \\ \mu(z-2) &= 0 \\ \mu &\geq 0 \end{cases}$$

Solutions of KKT system:

$$\begin{cases} x=-1/2,\ y=3,\ z=2,\ \lambda=-2,\ \mu=5 \\ x=-1/2,\ y=1/2,\ z=-1/2,\ \lambda=1/2,\ \mu=0 \end{cases}$$

Discussion of the stationary point: $x=-1/2,\ y=3,\ z=2,\ \lambda=-2,\ \mu=5.$

4

Solve the following recurrence

$$x_{k+1} = x_k - k, \qquad x_0 = 1$$
$$y_{k+1} = x_k + y_k + 1, \qquad y_0 = 1$$

$\mathcal{Z}-$transform:

$$\zeta x(\zeta) - \zeta = x(\zeta) - \zeta/(\zeta-1)^2$$

$$\zeta y(\zeta) - \zeta = x(\zeta) + \zeta/(\zeta-1)^2$$

Solution in $\mathcal{Z}$:

$$x(\zeta) = \frac{\zeta^2(\zeta-2)}{(\zeta-1)^3} = \frac{\zeta}{\zeta-1} - \frac{\zeta}{(\zeta-1)^3},$$

$$y(\zeta) = \frac{\zeta^2(\zeta^2-\zeta-1)}{(\zeta-1)^4} = \frac{\zeta}{\zeta-1} + \frac{2\zeta}{(\zeta-1)^2} - \frac{\zeta}{(\zeta-1)^4},$$

Solution in k

$$x_k = 1 - \frac{1}{2}k(k-1),$$

$$y_k = 1 + 2k - \binom{k}{3} = 1 + \frac{5}{3}k + \frac{k^2}{2} - \frac{k^3}{6},$$

5

Given the following system of ODE:

$$\begin{cases} x''(t) + x(t) = t \\ \\ y'(t) - x(t) - y(t) = -t \\ \\ x(0) = 0, \qquad y(0) = 1, \qquad x'(0) = A, \\ \end{cases}$$

Compute constant A in such a way $x(1) = 1$.

Laplace transform:

$$\begin{cases} s^2 x(s) - A + x(s) &= \dfrac{1}{s} \\ sy(s) - 1 - x(s) &= -\dfrac{1}{s^2} \end{cases}$$

Solution in s:

$$\begin{cases} x(s) &= \dfrac{As^2+1}{s^2(s^2+1)} = \dfrac{1}{s^2} + \dfrac{A-1}{s^2+1}, \\ y(s) &= \dfrac{A+s^2}{s^3 - s^2 + s - 1} = \dfrac{A+s^2}{(s-1)(s^2+1)} = \dfrac{A+1}{2(s-1)} + \dfrac{(1-A)(1+s)}{2(s^2+1)}, \end{cases}$$

Solution in t:

$$\begin{cases} x(t) &= t + (A-1)\sin t = t, \\ y(t) &= \dfrac{A+1}{2}e^t - \dfrac{A-1}{2}(\cos t + \sin t) = e^t, \end{cases}$$

Constant A

$$A = 1$$