Computational Methods for Mechatronics [140466] — 2015, June 8

Surname | | Name | | Mat. number | |

Signature ______________________

1

Compute the control (as a function of states and co-states) for the following optimal control problem

$$
\begin{cases}
\text{Minimize:} \quad \displaystyle\int_0^1 x(t)^2 + y(t)^2 \,\mathrm{d}t \\
\text{Integral constraint:} \quad \displaystyle\int_0^1 x(t)^2 \,\mathrm{d}t = 1 \\
x'(t) = y(t) \\
y'(t) = x(t) + u(t) \\
x(0) = 0, \quad x(1) = 0, \quad 0 \leq u(t) \leq 1
\end{cases}
$$

The Boundary Value Problem (BVP):
The optimal control $u(t)$:

2

Compute the control (as a function of states and co-states) for the following **free time** optimal control problem

$$\begin{cases} \text{Minimize:} \quad \int_0^T u(t)^2 \, \mathrm{d}t \\ x'(t) = y(t) + u(t) \\ y'(t) = x(t) \\ x(0) = 0, \quad x(T) = 1, \quad y(0) = 0 \end{cases}$$

The transformed (OCP):
The Boundary Value Problem (BVP):
The optimal control $u(t)$:

3

Study the following constrained minimization problem.

$$\text{minimize} \quad f(x,y,z) = x\,y^2 + z, \qquad z + y + x = 1, \qquad x \leq 3,$$

KKT system of first order condition:

$$\begin{cases} y^2 - \lambda + \mu &= 0 \\ 2xy - \lambda &= 0 \\ -\lambda + 1 &= 0 \\ z + y + x - 1 &= 0 \\ (3 - x)\mu &= 0 \end{cases}$$

Solutions of KKT system:

$$\begin{cases} x = 1/2,\ y = 1,\ z = -1/2,\ \lambda = 1,\ \mu = 0 \\ x = -1/2,\ y = -1,\ z = 5/2,\ \lambda = 1,\ \mu = 0 \\ x = 3,\ y = 1/6,\ z = -13/6,\ \lambda = 1,\ \mu = 35/36 \end{cases}$$

Discussion of the stationary point: $x = 3,\ y = 1/6,\ z = -13/6,\ \lambda = 1,\ \mu = 35/36.$

4

Solve the following recurrence

$$x_{k+2} = 2x_{k+1} - x_k + k, \qquad x_0 = 1, \quad x_1 = 0$$

$\mathcal{Z}-$transform:

$$\zeta^2(x(\zeta) - 1) = 2\zeta(x(\zeta) - 1) - x(\zeta) + \zeta/(\zeta - 1)^2$$

Solution in $\mathcal{Z}$:

$$x(\zeta) = \zeta\frac{\zeta^3 - 4\zeta^2 + 5\zeta - 1}{(\zeta - 1)^4}$$

Solution in k

$$x_k = \frac{1}{6}(k-1)(k^2 - 2k - 6)$$

5

Given the following system of ODE:

$$\begin{cases} x''(t) + y'(t) = -1 \\ \\ y'(t) - x'(t) = -2 \\ \\ x(0) = 0, \qquad y(0) = 0, \qquad x'(0) = A, \\ \end{cases}$$

Compute constant A in such a way $x(1) = 1$.

Laplace transform:

$$\begin{cases} s^2 x(s) - A + s y(s) &= -1/s \\ s y(s) - s x(s) &= -2/s \end{cases}$$

Solution in s:

$$\begin{cases} x(s) = \dfrac{As+1}{s^2(s+1)} = \dfrac{A-1}{s} + \dfrac{1-A}{s+1} + \dfrac{1}{s^2}, \\ \\ y(s) = \dfrac{As-2s-1}{s^2(s+1)} = \dfrac{A-1}{s} + \dfrac{1-A}{s+1} - \dfrac{1}{s^2}, \end{cases}$$

Constant A

$$A = 1$$

Solution in t:

$$\begin{cases} x(t) = -1 + A + t - \exp(-t)(A-1) = t, \\ \\ y(t) = -1 + A - t - \exp(-t)(A-1) = -t, \end{cases}$$