Computational Methods for Mechatronics [140466] — 2015, July 6

Surname | | Name | | Mat. number | |

Signature ______________________

1

Compute the control (as a function of states and co-states) for the following optimal control problem

$$
\begin{cases}
\text{Minimize:} \quad \displaystyle\int_0^1 x(t)y(t)\,\mathrm{d}t \\
\text{Integral constraint:} \quad \displaystyle\int_0^1 x(t)^2 + y(t)\,\mathrm{d}t = 1 \\
x'(t) = x(t) - y(t) \\
y'(t) = x(t) + u(t) \\
x(0) = 0, \quad x(1) = 0, \quad 0 \leq u(t) \leq 1
\end{cases}
$$

The Boundary Value Problem (BVP):
The optimal control $u(t)$:

2

Compute the control (as a function of states and co-states) for the following **free time** optimal control problem

$$\begin{cases} \text{Minimize:} \quad \int_0^T (x(t)^2 + 1)u(t)^2 \, \mathrm{d}t \\ x'(t) = x(t) + u(t) \\ y'(t) = x(t) \\ x(0) = 0, \quad x(T) = 1, \quad y(0) = 0 \end{cases}$$

The transformed (OCP):

The Boundary Value Problem (BVP):

The optimal control $u(t)$:

3

Study the following constrained minimization problem.

$$\text{minimize} \quad f(x,y,z) = x + y^2 + z, \qquad x\,z + x = 1, \qquad z + 3 \geq 0,$$

KKT system of first order condition:

$$\begin{cases} 1-(z+1)\lambda &= 0 \\ 2y &= 0 \\ 1-\lambda x-\mu &= 0 \\ xz+x-1 &= 0 \\ (3+z)\mu &= 0 \end{cases}$$

Solutions of KKT system:

$$\begin{cases} x=-1/2,\ y=0,\ z=-3,\ \lambda=-1/2,\ \mu=3/4 \\ x=1,\ y=0,\ z=0,\ \lambda=1,\ \mu=0 \\ x=-1,\ y=0,\ z=-2,\ \lambda=-1,\ \mu=0 \end{cases}$$

Discussion of the stationary point: $x=-1/2,\ y=0,\ z=-3,\ \lambda=-1/2,\ \mu=3/4$.

4

Solve the following recurrence

$$x_{k+2} = x_{k+1} + 2x_k + k, \qquad x_0 = 1, \quad x_1 = 0$$

$\mathcal{Z}-$transform:

$$\zeta^2(x(\zeta) - 1) = \zeta x(\zeta) - \zeta + 2x(\zeta) + \frac{\zeta}{(\zeta - 1)^2}$$

Solution in $\mathcal{Z}$:

$$x(\zeta) = \frac{\zeta^2(\zeta^2 - 3\zeta + 3)}{(\zeta - 1)^2(\zeta + 1)(\zeta - 2)}$$

Solution in k

$$x_k = \frac{1}{3}2^{k+1} + \frac{7}{12}(-1)^k - \frac{1}{4} - \frac{k}{2}$$

5

Given the following system of ODE:

$$\begin{cases} x''(t) + y'(t) = 2t \\ \\ y'(t) + x'(t) = 2t + 1 \\ \\ x(0) = 0, \qquad y(0) = 0, \qquad x'(0) = A, \end{cases}$$

Compute constant A in such a way $x(1) = 1$.

Laplace transform:

$$\begin{cases} s^2 x(s) - A + s y(s) &= \dfrac{2}{s^2} \\ s y(s) + s x(s) &= \dfrac{s+2}{s^2} \end{cases}$$

Solution in s:

$$\begin{cases} x(s) &= \dfrac{As - 1}{s^2(s-1)} = \dfrac{1-A}{s} + \dfrac{A-1}{s-1} + \dfrac{1}{s^2}, \\ \\ y(s) &= \dfrac{(1-A)s^2 + 2s - 2}{s^3(s-1)} = \dfrac{A-1}{s} + \dfrac{1-A}{s-1} + \dfrac{2}{s^3}, \end{cases}$$

Constant A

$$A = 1$$

Solution in t:

$$\begin{cases} x(t) = 1 + t - A + \exp(-t)(A-1) = t, \\ \\ y(t) = t^2 - 1 + A - \exp(-t)(A-1) = t^2, \end{cases}$$