Computational Methods for Mechatronics [140466] — 2016, January 13

Surname | | Name | | Mat. number | |

Signature ______________________

1

Compute the control (as a function of states and co-states) for the following optimal control problem

$$\begin{cases} \text{Minimize:} \quad \displaystyle\int_0^1 x(t)^2 - y(t)\,\mathrm{d}t \\ \text{Integral constraint:} \quad \displaystyle\int_0^1 x(t)\,\mathrm{d}t = 1 \\ x'(t) = x(t) - y(t) \\ y'(t) = x(t) + y(t)u(t) \\ x(0) = 0, \quad y(1) = 0, \quad -1 \leq u(t) \leq 10 \end{cases}$$

The Boundary Value Problem (BVP):
The optimal control $u(t)$:

2

Compute the control (as a function of states and co-states) for the following **free time** optimal control problem

$$\begin{cases} \text{Minimize:} \quad \int_0^T (x(t)^2+1)u(t)^2\,\mathrm{d}t \\ x'(t) = y(t) + u(t) \\ y'(t) = x(t) \\ x(0)=0, \quad y(0)=0, \quad y(T)=1, \end{cases}$$

The transformed (OCP):
The Boundary Value Problem (BVP):
The optimal control $u(t)$:

3

Study the following constrained minimization problem.

$$\text{minimize} \quad f(x,y,z) = x^2 - y + z, \qquad y - x\,z = 1, \qquad z + 1 \geq 0,$$

KKT system of first order condition:

Solutions of KKT system:

Discussion of the stationary point: $x = -1/2,\ y = 3/2,\ z = -1,\ \lambda = -1,\ \mu = 3/2.$

4

Solve the following recurrence

$$x_{k+2} = x_k - k(k-1), \qquad x_0 = 1, \quad x_1 = 0$$

$\mathcal{Z}-$transform:

Solution in $\mathcal{Z}$:

Solution in k

5

Given the following system of ODE:

$$\begin{cases} x''(t) + y'(t) = e^t + te^t \\ \\ y'(t) + x'(t) = 1 + e^t + te^t \\ \\ x(0) = 0, \qquad y(0) = 0, \qquad x'(0) = A, \\ \end{cases}$$

Compute constant A in such a way $x(1) = 1$.

Laplace transform:

Solution in s:

Constant A

Solution in t: