Computational Methods for Mechatronics [140466] — 2016, January 27

Surname		Name		Mat. number	

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1

Compute the control (as a function of states and co-states) for the following optimal control problem

$$
\begin{cases}
\text{Minimize:} \quad \displaystyle\int_0^1 x(t)y(t)\,\mathrm{d}t \\
\text{Integral constraint:} \quad \displaystyle\int_0^1 x^2(t) + y(t)\,\mathrm{d}t = 1 \\
x'(t) = x(t) + y(t) \\
y'(t) = x(t) + u(t) \\
x(0) = 1, \quad y(1) = 2, \quad -2 \leq u(t) \leq 3
\end{cases}
$$

The Boundary Value Problem (BVP):
The optimal control $u(t)$:

2

Compute the control (as a function of states and co-states) for the following **free time** optimal control problem

$$\begin{cases} \text{Minimize:} \quad \int_0^T (x(t)^2 + y(t)^2) u(t)^2 \,\mathrm{d}t \\ x'(t) = y(t) + x(t)u(t) \\ y'(t) = y(t) + u(t) \\ x(0) = 1, \quad y(0) = 2, \quad y(T) = 3, \end{cases}$$

The transformed (OCP):

The Boundary Value Problem (BVP):

The optimal control $u(t)$:

3

Study the following constrained minimization problem.

$$\text{minimize} \quad f(x,y,z) = xy + z^2 + 2, \qquad z + y = 1, \qquad x \geq y,$$

KKT system of first order condition:

Solutions of KKT system:

Discussion of the stationary point: $x = 2,\ y = 0,\ z = 1,\ \lambda = 2,\ \mu = 0.$

4

Solve the following recurrence

$$x_{k+2} = -x_k - k, \qquad x_0 = 0, \quad x_1 = 0$$

$\mathcal{Z}-$transform:

Solution in $\mathcal{Z}$:

Solution in k

5

Given the following system of ODE:

$$\begin{cases} x''(t) - y'(t) = 0 \\ y''(t) + x'(t) = 0 \\ x(0) = 0, \qquad y(0) = 0, \qquad x'(0) = 1, \qquad y'(0) = A, \end{cases}$$

Compute constant A in such a way $x(\pi/2) = 2$.

Laplace transform:

Solution in s:

Constant A

Solution in t: