## Computational Methods for Mechatronics [140466] - 2016, January 27

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## 1

Compute the control (as a function of states and co-states) for the following optimal control problem

$$\begin{cases} \text{Minimize:} \quad \int_{0}^{1} x(t)y(t) \, \mathrm{d}t \\\\ \text{Integral constraint:} \quad \int_{0}^{1} x^{2}(t) + y(t) \, \mathrm{d}t = 1 \\\\ x'(t) = x(t) + y(t) \\\\ y'(t) = x(t) + u(t) \\\\ x(0) = 1, \quad y(1) = 2, \quad -2 \le u(t) \le 3 \end{cases}$$

The Boundary Value Problem (BVP):

The optimal control u(t):

## 2

Compute the control (as a function of states and co-states) for the following  $\mathbf{free}\ \mathbf{time}$  optimal control problem

$$\begin{cases} \text{Minimize:} & \int_0^T (x(t)^2 + y(t)^2) u(t)^2 \, \mathrm{d}t \\ x'(t) &= y(t) + x(t) u(t) \\ y'(t) &= y(t) + u(t) \\ x(0) &= 1, \quad y(0) = 2, \quad y(T) = 3, \end{cases}$$

The transformed (OCP):

The Boundary Value Problem (BVP):

The optimal control u(t):

Study the following constrained minimization problem.

minimize  $f(x, y, z) = xy + z^2 + 2, \qquad z + y = 1, \qquad x \ge y,$ 

KKT system of first order condition:

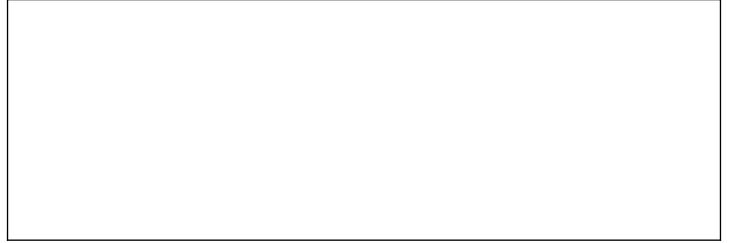
Solutions of KKT system:

Discussion of the stationary point:  $x = 2, y = 0, z = 1, \lambda = 2, \mu = 0.$ 

Solve the following recurrence

$$x_{k+2} = -x_k - k, \qquad x_0 = 0, \quad x_1 = 0$$

 $\mathcal{Z}$ -transform:



## Solution in $\mathcal{Z}$ :

Solution in k

Given the following system of ODE:

$$\begin{cases} x''(t) - y'(t) = 0\\ y''(t) + x'(t) = 0\\ x(0) = 0, \qquad y(0) = 0, \qquad x'(0) = 1, \qquad y'(0) = A, \end{cases}$$

Compute constant A in such a way  $x(\pi/2) = 2$ .

Laplace transform:

Solution in s:

Constant A

Solution in t: