Fattorizzazione LU con pivoting parziale

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```
> with(LinearAlgebra) :
```

Matrice da fattorizzare

```
> A := <<0,1,1,1>|<2,3,1,0>|<1,0,0,0>|<2,0,1,-1>>;
```

$$A := \begin{bmatrix} 0 & 2 & 1 & 2 \\ 1 & 3 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & -1 \end{bmatrix} \tag{1.1}$$

Prime matrici di scambio e di Frobenius

```
> # scambio la prima con la seconda riga
  S1 := <<0,1,0,0>|<1,0,0,0>|<0,0,1,0>|<0,0,0,1>>;
```

$$S1 := \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2.1}$$

```
> # effetto della moltiplicazione
  S1.A ;
```

$$\begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & 2 & 1 & 2 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & -1 \end{bmatrix} \tag{2.2}$$

```
> L1 := <<1,0,-1,-1>|<0,1,0,0>|<0,0,1,0>|<0,0,0,1>>;
```

$$L1 := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix} \tag{2.3}$$

```
> # effetto della moltiplicazione
  L1.S1.A ;
```

(2.4)

$$\begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & 2 & 1 & 2 \\ 0 & -2 & 0 & 1 \\ 0 & -3 & 0 & -1 \end{bmatrix} \quad (2.4)$$

Seconde matrici di scambio e di Frobenius

```
> # scambio la seconda con la quarta riga
  S2 := <<1,0,0,0>|<0,0,0,1>|<0,0,1,0>|<0,1,0,0>>;
```

$$S2 := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad (3.1)$$

```
> # effetto della moltiplicazione
  S2.L1.S1.A ;
```

$$\begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & -3 & 0 & -1 \\ 0 & -2 & 0 & 1 \\ 0 & 2 & 1 & 2 \end{bmatrix} \quad (3.2)$$

```
> L2 := <<1,0,0,0>|<0,1,-2/3,2/3>|<0,0,1,0>|<0,0,0,1>>;
```

$$L2 := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -\frac{2}{3} & 1 & 0 \\ 0 & \frac{2}{3} & 0 & 1 \end{bmatrix} \quad (3.3)$$

```
> # effetto della moltiplicazione
  L2.S2.L1.S1.A ;
```

$$\begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & -3 & 0 & -1 \\ 0 & 0 & 0 & \frac{5}{3} \\ 0 & 0 & 1 & \frac{4}{3} \end{bmatrix} \quad (3.4)$$

Seconde matrici di scambio e di Frobenius

```
> # scambio la terza con la quarta riga
  S3 := <<1,0,0,0>|<0,1,0,0>|<0,0,0,1>|<0,0,1,0>>;
```

$$S3 := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad (4.1)$$

```
> # effetto della moltiplicazione
  S3.L2.S2.L1.S1.A ;
```

$$\begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & -3 & 0 & -1 \\ 0 & 0 & 1 & \frac{4}{3} \\ 0 & 0 & 0 & \frac{5}{3} \end{bmatrix} \quad (4.2)$$

```
> L3 := <<1,0,0,0>|<0,1,0,0>|<0,0,1,0>|<0,0,0,1>>;
```

$$L3 := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4.3)$$

```
> # effetto della moltiplicazione
  L3.S3.L2.S2.L1.S1.A ;
```

$$\begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & -3 & 0 & -1 \\ 0 & 0 & 1 & \frac{4}{3} \\ 0 & 0 & 0 & \frac{5}{3} \end{bmatrix} \quad (4.4)$$

Fattorizzazione LU

```
> U := L3.S3.L2.S2.L1.S1.A ;
```

$$U := \begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & -3 & 0 & -1 \\ 0 & 0 & 1 & \frac{4}{3} \\ 0 & 0 & 0 & \frac{5}{3} \end{bmatrix} \quad (5.1)$$

```
> # osserviamo che
  S3.S3 ;
```

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (5.2)$$

```
> # inoltre vale l'identita`
  L3.S3.L2.S2.L1.S1 = L3.(S3.L2.S3).S3.S2.L1.S1 ;
```

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 \\ 1 & -\frac{2}{3} & 0 & \frac{2}{3} \\ 0 & -\frac{1}{3} & 1 & -\frac{2}{3} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 \\ 1 & -\frac{2}{3} & 0 & \frac{2}{3} \\ 0 & -\frac{1}{3} & 1 & -\frac{2}{3} \end{bmatrix} \quad (5.3)$$

```
> L3.(S3.L2.S3).S3.S2.L1.S1 = L3.(S3.L2.S3).(S3.S2.L1.S2.S3).S3.
  S2.S1 ;
```

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 \\ 1 & -\frac{2}{3} & 0 & \frac{2}{3} \\ 0 & -\frac{1}{3} & 1 & -\frac{2}{3} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 \\ 1 & -\frac{2}{3} & 0 & \frac{2}{3} \\ 0 & -\frac{1}{3} & 1 & -\frac{2}{3} \end{bmatrix} \quad (5.4)$$

```
> # quindi
  LL2 := (S3.L2.S3) ;
  LL1 := (S3.S2.L1.S2.S3) ;
```

$$LL2 := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & \frac{2}{3} & 1 & 0 \\ 0 & -\frac{2}{3} & 0 & 1 \end{bmatrix} \quad (5.5)$$

$$LL1 := \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix}$$

```
> # ponendo
```

```
L := (L3.LL2.LL1)^(-1) ;
P := S3.S2.S1 ;
```

$$L := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & -\frac{2}{3} & 1 & 0 \\ 1 & \frac{2}{3} & 0 & 1 \end{bmatrix} \quad (5.6)$$

$$P := \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

```
> L.U=P.A ;
```

$$\begin{bmatrix} 1 & 3 & 0 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 2 & 1 & 2 \\ 1 & 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 2 & 1 & 2 \\ 1 & 1 & 0 & 1 \end{bmatrix} \quad (5.7)$$