

# Prontuario per l'esame di Calcolo Numerico (E.Bertolazzi)

- $$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad x_{n+1} = \frac{f(x_n)x_{n-1} - f(x_{n-1})x_n}{f(x_n) - f(x_{n-1})}$$
- $$f(x) - P(x) = \frac{f^{(n+1)}(\zeta)}{(n+1)!} (x-x_0)(x-x_1)\cdots(x-x_n) \quad \left[ \begin{array}{l} P(x) = y_0L_0(x) + y_1L_1(x) + \cdots + y_nL_n(x) \\ L_k(x) = \frac{(x-x_0)\cdots(x-x_{k-1})(x-x_{k+1})\cdots(x-x_n)}{(x_k-x_0)\cdots(x_k-x_{k-1})(x_k-x_{k+1})\cdots(x_k-x_n)} \end{array} \right.$$
- $$P(x) = f(x_0) + f[x_0, x_1](x-x_0) + \cdots + f[x_0, x_1, x_2, \dots, x_n](x-x_0)(x-x_1)\cdots(x-x_{n-1})$$
- $$f[x_i] = f(x_i), \quad f[x_i, x_{i+1}] = \frac{f[x_i] - f[x_{i+1}]}{x_i - x_{i+1}}, \quad f[x_i, x_{i+1}, \dots, x_{i+k+1}] = \frac{f[x_i, x_{i+1}, \dots, x_{i+k}] - f[x_{i+1}, x_{i+2}, \dots, x_{i+k+1}]}{x_i - x_{i+k+1}}$$
- $$\int_a^b f(x)dx = \frac{h}{2} \left[ f(a) + f(b) + 2 \sum_{i=1}^{n-1} f(x_i) \right] - \frac{(b-a)^3}{12n^2} f''(\zeta)$$
- $$\int_a^b f(x)dx = \frac{h}{3} [f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 4f(x_{n-1}) + f(x_n)] - \frac{(b-a)^5}{180n^4} f^{(4)}(\zeta)$$
- $$\int_a^b f(x)dx \approx \frac{b-a}{2} \sum_{i=1}^n w_i f\left(\frac{a+b}{2} + x_i \frac{b-a}{2}\right)$$

$n$	$x_k$	$w_k$
2	-0.57735	1
	0	1
	+0.57735	1

$n$	$x_k$	$w_k$
3	-0.77460	0.55555555
	0	0.88888888
	+0.77460	0.55555555
- $$\frac{f(x+h) - f(x)}{h} = f'(x) + \frac{h}{2} f''(\zeta) \quad \frac{f(x+h) - f(x-h)}{2h} = f'(x) + \frac{h^2}{6} f'''(\zeta)$$
- $$\frac{f(x+h) - 2f(x) + f(x-h)}{h^2} = f''(x) + \frac{h^2}{12} f^{(4)}(\zeta)$$
- |   |   |  |
|---|---|--|
| $\begin{cases} y_0 = y(a) \\ \eta_{k+1} = y_k + hf(x_k, y_k) \\ y_{k+1} = y_k + \frac{h}{2} [f(x_k, y_k) + f(x_{k+1}, \eta_{k+1})] \end{cases}$ | $\begin{cases} y_0 = y(a) \\ y_{k+1/2} = y_k + \frac{h}{2} f(x_k, y_k) \\ y_{k+1} = y_k + hf(x_{k+1/2}, y_{k+1/2}) \end{cases}$ | $\begin{cases} y_0 = y(a) \\ y_{k+1} = y_k + hf(x_k, y_k) \end{cases}$ |
|---|---|--|
- |  |     |     |     |  |     |   |     |  |   |   |   |   |  |     |     |     |  |  |     |     |   |  |
|--|-----|-----|-----|--|-----|---|-----|--|---|---|---|---|--|-----|-----|-----|--|--|-----|-----|---|--|
| <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="padding: 5px;">1/2</td> <td style="padding: 5px;">1/2</td> <td style="padding: 5px;"></td> <td style="padding: 5px;"></td> </tr> <tr> <td style="padding: 5px;">1/2</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">1/2</td> <td style="padding: 5px;"></td> </tr> <tr> <td style="padding: 5px;">1</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">1</td> </tr> <tr> <td style="padding: 5px;"></td> <td style="padding: 5px;">1/6</td> <td style="padding: 5px;">1/3</td> <td style="padding: 5px;">1/3</td> </tr> <tr> <td style="padding: 5px;"></td> <td style="padding: 5px;"></td> <td style="padding: 5px;">1/3</td> <td style="padding: 5px;">1/6</td> </tr> </table> | 1/2 | 1/2 |     |  | 1/2 | 0 | 1/2 |  | 1 | 0 | 0 | 1 |  | 1/6 | 1/3 | 1/3 |  |  | 1/3 | 1/6 | $\begin{cases} y_0 = y(a) \\ K_s = hf\left(x_k + h\alpha_s, y_k + \sum_{j=1}^{s-1} \beta_{s,j} K_j\right) \quad s=1,2,\dots,n \\ y_{k+1} = y_k + \omega_1 K_1 + \omega_2 K_2 + \cdots + \omega_n K_n \end{cases}$ |  |
| 1/2  | 1/2 |     |     |  |     |   |     |  |   |   |   |   |  |     |     |     |  |  |     |     |   |  |
| 1/2  | 0   | 1/2 |     |  |     |   |     |  |   |   |   |   |  |     |     |     |  |  |     |     |   |  |
| 1  | 0   | 0   | 1   |  |     |   |     |  |   |   |   |   |  |     |     |     |  |  |     |     |   |  |
|  | 1/6 | 1/3 | 1/3 |  |     |   |     |  |   |   |   |   |  |     |     |     |  |  |     |     |   |  |
|  |     | 1/3 | 1/6 |  |     |   |     |  |   |   |   |   |  |     |     |     |  |  |     |     |   |  |
- $$y_{n+1} = \sum_{j=-k}^0 a_j y_{n+j} + h \sum_{j=-k}^1 b_j f_{n+j}, \quad f_i = f(x_i, y_i)$$
- $$(1) \sum_{j=-k}^0 a_j = 1 \quad (2) \sum_{j=-k}^0 a_j j^s + \sum_{j=-k}^1 b_j j^{s-1} = 1, \quad s=1,2,3,\dots \quad (\text{nota } j^0 = 1)$$
- $$\begin{cases} y_0 = y(a), \quad y_n = y(b) \\ \frac{y_{k+1} - 2y_k + y_{k-1}}{h^2} + p_k \frac{y_{k+1} - y_{k-1}}{2h} + q_k y_k = r_k \quad k=1,2,\dots,n-1 \end{cases}$$

- $$\begin{cases} y_0 = y(a) \\ y_n = y(b) \\ \frac{y_{k+1} - 2y_k + y_{k-1}}{h^2} + p_k^+ \frac{y_{k+1} - y_k}{h} + p_k^- \frac{y_k - y_{k-1}}{h} + q_k y_k = r_k \quad k = 1, 2, \dots, n-1 \end{cases}$$

- $$\mathbf{P} \mathbf{x}^{k+1} = \mathbf{b} - \mathbf{Q} \mathbf{x}^k$$

- $$\mathbf{D} \mathbf{x}^{k+1} = \mathbf{b} - (\mathbf{L} + \mathbf{U}) \mathbf{x}^k, \quad x_i^{k+1} = x_i^k + \frac{1}{A_{ii}} \left[ b_i - \sum_{j=1}^n A_{i,j} x_j^k \right],$$

- $$(\mathbf{D} + \mathbf{L}) \mathbf{x}^{k+1} = \mathbf{b} - \mathbf{U} \mathbf{x}^k, \quad x_i^{k+1} = \frac{1}{A_{ii}} \left[ b_i - \sum_{j=1}^{i-1} A_{i,j} x_j^{k+1} - \sum_{j=i+1}^n A_{i,j} x_j^k \right],$$

- $$\left( \frac{\mathbf{D}}{\omega} + \mathbf{L} \right) \mathbf{x}^{k+1} = \mathbf{b} - \left( \frac{\omega-1}{\omega} \mathbf{D} + \mathbf{U} \right) \mathbf{x}^k, \quad x_i^{k+1} = x_i^k + \frac{\omega}{A_{ii}} \left[ b_i - \sum_{j=1}^{i-1} A_{i,j} x_j^{k+1} - \sum_{j=i}^n A_{i,j} x_j^k \right],$$

- $$\begin{cases} \alpha = (\mathbf{r}^n)^T \mathbf{r}^n / (\mathbf{r}^n)^T \mathbf{A} \mathbf{r}^n \\ \mathbf{x}^{n+1} = \mathbf{x}^n + \alpha \mathbf{r}^n \\ \mathbf{r}^{n+1} = \mathbf{r}^n - \alpha \mathbf{A} \mathbf{r}^n \end{cases} \quad \begin{cases} \alpha = (\mathbf{r}^k)^T \mathbf{r}^k / (\mathbf{p}^k)^T \mathbf{A} \mathbf{p}^k \\ \mathbf{x}^{k+1} = \mathbf{x}^k + \alpha \mathbf{p}^k \\ \mathbf{r}^{k+1} = \mathbf{r}^k - \alpha \mathbf{A} \mathbf{p}^k \\ \beta = (\mathbf{r}^{k+1})^T \mathbf{r}^{k+1} / (\mathbf{r}^k)^T \mathbf{r}^k \text{ oppure } \beta = -(\mathbf{r}^{k+1})^T \mathbf{A} \mathbf{p}^k / (\mathbf{p}^k)^T \mathbf{A} \mathbf{p}^k \\ \mathbf{p}^{k+1} = \mathbf{r}^{k+1} + \beta \mathbf{p}^k \end{cases}$$

- $$\begin{cases} \mathbf{w}^{k+1} = \mathbf{A} \mathbf{u}^k \\ \mathbf{u}^{k+1} = \mathbf{w}^{k+1} / \|\mathbf{w}^{k+1}\| \\ \lambda_{\max} \approx (\mathbf{u}^k)^T \mathbf{A} \mathbf{u}^k / (\mathbf{u}^k)^T \mathbf{u}^k = (\mathbf{u}^k)^T \mathbf{w}^{k+1} / (\mathbf{u}^k)^T \mathbf{u}^k \end{cases} \quad \begin{cases} \mathbf{w}^{k+1} = \mathbf{A}^{-1} \mathbf{u}^k \\ \mathbf{u}^{k+1} = \mathbf{w}^{k+1} / \|\mathbf{w}^{k+1}\| \\ \lambda_{\min} \approx (\mathbf{u}^k)^T \mathbf{A}^{-1} \mathbf{u}^k / (\mathbf{u}^k)^T \mathbf{u}^k = (\mathbf{u}^k)^T \mathbf{w}^{k+1} / (\mathbf{u}^k)^T \mathbf{u}^k \end{cases}$$

- $$\begin{pmatrix} n+1 & \sum_{i=0}^n x_i \\ \sum_{i=0}^n x_i & \sum_{i=0}^n x_i^2 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} \sum_{i=0}^n y_i \\ \sum_{i=0}^n x_i y_i \end{pmatrix} \quad \begin{pmatrix} n+1 & \sum_{i=0}^n x_i & \sum_{i=0}^n x_i^2 \\ \sum_{i=0}^n x_i & \sum_{i=0}^n x_i^2 & \sum_{i=0}^n x_i^3 \\ \sum_{i=0}^n x_i^2 & \sum_{i=0}^n x_i^3 & \sum_{i=0}^n x_i^4 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} \sum_{i=0}^n y_i \\ \sum_{i=0}^n x_i y_i \\ \sum_{i=0}^n x_i^2 y_i \end{pmatrix}$$

- $$h_{k-1/2} = x_k - x_{k-1},$$

$$S_k(x) = y_{k-1} + \left( \frac{y_k - y_{k-1}}{h_{k-1/2}} - h_{k-1/2} \frac{M_k + 2M_{k-1}}{6} \right) (x - x_{k-1}) + \frac{M_{k-1}}{2} (x - x_k)^2 + \frac{M_k - M_{k-1}}{6h_{k-1/2}} (x - x_{k-1})^3$$

$$\frac{h_{k-1/2}}{h_{k-1/2} + h_{k+1/2}} M_{k-1} + 2M_k + \frac{h_{k+1/2}}{h_{k-1/2} + h_{k+1/2}} M_{k+1} = \frac{6}{h_{k-1/2} + h_{k+1/2}} \left( \frac{y_{k+1} - y_k}{h_{k+1/2}} - \frac{y_k - y_{k-1}}{h_{k-1/2}} \right)$$