Prontuario per l'esame di Calcolo Numerico (E.Bertolazzi)

- $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}, \qquad x_{n+1} = \dfrac{f(x_n)x_{n-1} - f(x_{n-1})x_n}{f(x_n) - f(x_{n-1})}$
- $f(x) - p(x) = \dfrac{f^{(n+1)}(\zeta)}{(n+1)!}(x-x_0)(x-x_1)\cdots(x-x_n), \quad \left[\begin{array}{l} p(x) = y_0 L_0(x) + y_1 L_1(x) + \cdots + y_n L_n(x) \\ L_k(x) = \prod_{i=0,i\neq k}^{n}(x-x_i) \Big/ \prod_{i=0,i\neq k}^{n}(x_k - x_i) \end{array}\right.$
- $p(x) = f(x_0) + f[x_0,x_1](x-x_0) + \cdots + f[x_0,x_1,x_2,\ldots,x_n](x-x_0)(x-x_1)\cdots(x-x_{n-1})$

  $f[x_i] = f(x_i), \quad f[x_i,x_{i+1}] = \dfrac{f[x_i] - f[x_{i+1}]}{x_i - x_{i+1}},$

  $f[x_i,x_{i+1},\ldots,x_{i+k+1}] = \dfrac{f[x_i,x_{i+1},\ldots,x_{i+k}] - f[x_{i+1},x_{i+2},\ldots,x_{i+k+1}]}{x_i - x_{i+k+1}},$
- $\displaystyle\int_a^b f(x)dx = \frac{h}{2}\left[f(a) + f(b) + 2\sum_{i=1}^{n-1} f(x_i)\right] - \frac{(b-a)^3}{12n^2} f''(\zeta)$
- $\displaystyle\int_a^b f(x)dx = \frac{h}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 4f(x_{n-1}) + f(x_n)\right] - \frac{(b-a)^5}{180n^4} f^{(4)}(\zeta)$
- $\displaystyle\int_a^b f(x)dx \approx \frac{b-a}{2}\sum_{i=1}^{n} w_i f\left(\frac{a+b}{2} + x_i\frac{b-a}{2}\right)$

n	x_k	w_k
2	–0.57735	1
	+0.57735	1

n	x_k	w_k
3	–0.77460	0.55555555
	0	0.88888888
	+0.77460	0.55555555

- $\dfrac{f(x+h) - f(x)}{h} = f'(x) + \dfrac{h}{2} f''(\zeta) \quad \dfrac{f(x+h) - f(x-h)}{2h} = f'(x) + \dfrac{h^2}{6} f'''(\zeta)$
- $\dfrac{f(x+h) - 2f(x) + f(x-h)}{h^2} = f''(x) + \dfrac{h^2}{12} f^{(4)}(\zeta)$
- $\begin{cases} y_0 = y(a) \\ \eta_{k+1} = y_k + hf(x_k,y_k) \\ y_{k+1} = y_k + \frac{h}{2}\left[f(x_k,y_k) + f(x_{k+1},\eta_{k+1})\right] \end{cases} \quad \begin{cases} y_0 = y(a) \\ y_{k+1/2} = y_k + \frac{h}{2} f(x_k,y_k) \\ y_{k+1} = y_k + hf(x_{k+1/2},y_{k+1/2}) \end{cases} \quad \begin{cases} y_0 = y(a) \\ y_{k+1} = y_k + hf(x_k,y_k) \end{cases}$
- $\begin{array}{c|cccc} 0 & & & & \\ 1/2 & 1/2 & & & \\ 1/2 & 0 & 1/2 & & \\ 1 & 0 & 0 & 1 & \\ \hline & 1/6 & 1/3 & 1/3 & 1/6 \end{array} \qquad \begin{cases} y_0 = y(a) \\ K_s = hf\left(x_k + h\alpha_s,\ y_k + \sum_{j=1}^{s-1}\beta_{s,j}K_j\right) & s = 1,2,\ldots,n \\ y_{k+1} = y_k + \omega_1 K_1 + \omega_2 K_2 + \cdots + \omega_n K_n \end{cases}$
- $\left| \begin{array}{l} y_{k+1} = \sum_{j=-p}^{0} a_j y_{k+j} + h\sum_{j=-p}^{1} b_j f_{k+j}, \quad f_i = f(x_i,y_i) \\ (1) \sum_{j=-p}^{0} a_j = 1 \qquad \sum_{j=-p}^{0} a_j j + \sum_{j=-p}^{0} b_j = 1, \\ (2) \sum_{j=-p}^{0} a_j j^s + s\sum_{j=-p}^{0} b_j j^{s-1} = 1, \quad s = 2,3,\ldots \end{array}\right.$
- $\begin{cases} y_0 = y(a), \quad y_n = y(b) \\ \dfrac{y_{k+1} - 2y_k + y_{k-1}}{h^2} + p_k\dfrac{y_{k+1} - y_{k-1}}{2h} + q_k y_k = r_k \quad k = 1,2,\ldots,n-1 \end{cases}$

- $$\begin{cases} y_0 = y(a), \quad y_n = y(b) \\ \dfrac{y_{k+1} - 2y_k + y_{k-1}}{h^2} + p_k^+ \dfrac{y_{k+1} - y_k}{h} + p_k^- \dfrac{y_k - y_{k-1}}{h} + q_k y_k = r_k \quad k = 1,2,\ldots,n-1 \end{cases}$$
- $$\mathbf{P}\,\mathbf{x}^{k+1} = \mathbf{b} - \mathbf{Q}\,\mathbf{x}^k$$
- $$\mathbf{D}\mathbf{x}^{k+1} = \mathbf{b} - (\mathbf{L} + \mathbf{U})\,\mathbf{x}^k, \quad x_i^{k+1} = x_i^k + \frac{1}{A_{ii}}\left[b_k - \sum_{j=1}^{n} A_{i,j} x_j^k\right],$$
- $$(\mathbf{D} + \mathbf{L})\mathbf{x}^{k+1} = \mathbf{b} - \mathbf{U}\,\mathbf{x}^k, \quad x_i^{k+1} = \frac{1}{A_{ii}}\left[b_i - \sum_{j=1}^{i-1} A_{i,j} x_j^{k+1} - \sum_{j=i+1}^{n} A_{i,j} x_j^k\right],$$
- $$\left(\frac{\mathbf{D}}{\omega} + \mathbf{L}\right)\mathbf{x}^{k+1} = \mathbf{b} - \left(\frac{\omega - 1}{\omega}\mathbf{D} + \mathbf{U}\right)\mathbf{x}^k, \quad x_i^{k+1} = x_i^k + \frac{\omega}{A_{ii}}\left[b_i - \sum_{j=1}^{i-1} A_{i,j} x_j^{k+1} - \sum_{j=i}^{n} A_{i,j} x_j^k\right],$$
- $$\begin{cases} \alpha = (\mathbf{r}^n)^T \mathbf{r}^n \big/ (\mathbf{r}^n)^T \mathbf{A}\mathbf{r}^n \\ \mathbf{x}^{n+1} = \mathbf{x}^n + \alpha \mathbf{r}^n \\ \mathbf{r}^{n+1} = \mathbf{r}^n - \alpha \mathbf{A}\mathbf{r}^n \end{cases} \qquad \begin{cases} \alpha = \left(\mathbf{r}^k\right)^T \mathbf{r}^k \Big/ \left(\mathbf{p}^k\right)^T \mathbf{A}\mathbf{p}^k \\ \mathbf{x}^{k+1} = \mathbf{x}^k + \alpha\,\mathbf{p}^k \\ \mathbf{r}^{k+1} = \mathbf{r}^k - \alpha \mathbf{A}\mathbf{p}^k \\ \beta = \left(\mathbf{r}^{k+1}\right)^T \mathbf{r}^{k+1} \Big/ \left(\mathbf{r}^k\right)^T \mathbf{r}^k \text{ oppure } \beta = -\left(\mathbf{r}^{k+1}\right)^T \mathbf{A}\mathbf{p}^k \Big/ \left(\mathbf{p}^k\right)^T \mathbf{A}\mathbf{p}^k \\ \mathbf{p}^{k+1} = \mathbf{r}^{k+1} + \beta\,\mathbf{p}^k \end{cases}$$
- $$\left[\begin{array}{l} \mathbf{w}^{k+1} = \mathbf{A}\mathbf{u}^k \\ \mathbf{u}^{k+1} = \mathbf{w}^{k+1} \big/ \left\|\mathbf{w}^{k+1}\right\| \\ \lambda_{\max} \approx \mathbf{w}^{k+1} \bullet \mathbf{u}^k \big/ \mathbf{u}^k \bullet \mathbf{u}^k \end{array}\right. \qquad \left[\begin{array}{l} \mathbf{w}^{k+1} = (\mathbf{A} - \mu \mathbf{I})^{-1} \mathbf{u}^k \\ \mathbf{u}^{k+1} = \mathbf{w}^{k+1} \big/ \left\|\mathbf{w}^{k+1}\right\| \\ \lambda_s \approx 1 - \mathbf{u}^k \bullet \mathbf{u}^k \big/ \mathbf{w}^{k+1} \bullet \mathbf{u}^k \end{array}\right. \qquad \mu \approx \lambda_s$$
- $$\begin{pmatrix} n+1 & \sum_{i=0}^{n} x_i \\ \sum_{i=0}^{n} x_i & \sum_{i=0}^{n} x_i^2 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} \sum_{i=0}^{n} y_i \\ \sum_{i=0}^{n} x_i y_i \end{pmatrix} \qquad \begin{pmatrix} n+1 & \sum_{i=0}^{n} x_i & \sum_{i=0}^{n} x_i^2 \\ \sum_{i=0}^{n} x_i & \sum_{i=0}^{n} x_i^2 & \sum_{i=0}^{n} x_i^3 \\ \sum_{i=0}^{n} x_i^2 & \sum_{i=0}^{n} x_i^3 & \sum_{i=0}^{n} x_i^4 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} \sum_{i=0}^{n} y_i \\ \sum_{i=0}^{n} x_i y_i \\ \sum_{i=0}^{n} x_i^2 y_i \end{pmatrix}$$
- $$S_k(x) = y_{k-1} + \left(\frac{y_k - y_{k-1}}{x_k - x_{k-1}} - (x_k - x_{k-1})\frac{M_k + 2M_{k-1}}{6}\right)(x - x_{k-1}) + \frac{M_{k-1}}{2}(x - x_{k-1})^2 + \frac{M_k - M_{k-1}}{6(x_k - x_{k-1})}(x - x_{k-1})^3$$
- $$\frac{x_k - x_{k-1}}{x_{k+1} - x_{k-1}} M_{k-1} + 2M_k + \frac{x_{k+1} - x_k}{x_{k+1} - x_{k-1}} M_{k+1} = \frac{6}{x_{k+1} - x_{k-1}}\left(\frac{y_{k+1} - y_k}{x_{k+1} - x_k} - \frac{y_k - y_{k-1}}{x_k - x_{k-1}}\right)$$