Lezione 21 marzo - Esempi con i metodi iterativi

Error, unexpected number

```
> restart:
> with(LinearAlgebra):
  with(plots):
```

Sistema lineare: Matrice dei coefficienti

```
> A := <<2,2,0>|<0,2,1>|<1,0,2>>;
```

$$A := \begin{bmatrix} 2 & 0 & 1 \\ 2 & 2 & 0 \\ 0 & 1 & 2 \end{bmatrix} \quad (1)$$

Soluzione esatta e termine noto:

```
> xe := <1,0,-2> ;
  b  := A.xe ;
```

$$xe := \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$$

$$b := \begin{bmatrix} 0 \\ 2 \\ -4 \end{bmatrix} \quad (2)$$

Estraggo parte sopra, sotto e diagonale (non in maniera elegante!)

```
> DD := Matrix(3,3):
  LL := Matrix(3,3):
  UU := Matrix(3,3):
  for i from 1 to 3 do
    for j from 1 to 3 do
      if i < j then
        UU[i,j] := A[i,j] ;
      elif i > j then
        LL[i,j] := A[i,j] ;
      else
        DD[i,j] := A[i,j] ;
      end;
    end;
  end;
```

Punto iniziale per i vari metodi iterativi

```
> x0 := <20,-10,1> ;
```

$$x0 := \begin{bmatrix} 20 \\ -10 \\ 1 \end{bmatrix} \quad (3)$$

```
> RS := [] ; # raggio spettrale
  N1 := [] ; # norma uno di Q P^(-1)
  NI := [] ; # norma infinito di Q P^(-1)
  N2 := [] ; # norma due di Q P^(-1)
```

$$RS := [\,]$$

$$N1 := [\ ]$$
$$NI := [\ ]$$
$$N2 := [\ ] \quad (4)$$

Metodo di Jacobi

Scrivo il metodo nella forma di splitting P-Q

```
> P := DD ;
  Q := -LL-UU ;
```

$$P := \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$
$$Q := \begin{bmatrix} 0 & 0 & -1 \\ -2 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} \quad (1.1)$$

Il metodo iterativo è P x[k+1] := Q x[k] + b

10 iterate, andamento del residuo e dell'errore

```
> r_jacobi := [] ; # vettore con norma residui
  e_jacobi := [] ; # vettore con norma errore
```

$$r\_jacobi := [\ ]$$
$$e\_jacobi := [\ ] \quad (1.2)$$

```
> x1 := P^(-1).(Q.x0+b) ; r1 := b - A.x1 :
  r_jacobi := [op(r_jacobi), norm(r1)];
  e_jacobi := [op(e_jacobi), norm(x1-xe)];
```

$$x1 := \begin{bmatrix} -\frac{1}{2} \\ -19 \\ 3 \end{bmatrix}$$
$$r\_jacobi := [41]$$
$$e\_jacobi := [19] \quad (1.3)$$

```
> x2 := P^(-1).(Q.x1+b) ; r2 := b - A.x2 ;
  r_jacobi := [op(r_jacobi), norm(r2)];
  e_jacobi := [op(e_jacobi), norm(x2-xe)];
```

$$x2 := \begin{bmatrix} -\frac{3}{2} \\ \frac{3}{2} \\ \frac{15}{2} \end{bmatrix}$$

$$r2 := \begin{bmatrix} -\frac{9}{2} \\ 2 \\ -\frac{41}{2} \end{bmatrix}$$

$$r\_jacobi := \left[41, \frac{41}{2}\right]$$

$$e\_jacobi := \left[19, \frac{19}{2}\right] \quad (1.4)$$

```
> x3 := P^(-1).(Q.x2+b) ; r3 := b - A.x3 ;
  r_jacobi := [op(r_jacobi), norm(r3)];
  e_jacobi := [op(e_jacobi), norm(x3-xe)];
```

$$x3 := \begin{bmatrix} -\frac{15}{4} \\ \frac{5}{2} \\ -\frac{11}{4} \end{bmatrix}$$

$$r3 := \begin{bmatrix} \frac{41}{4} \\ \frac{9}{2} \\ -1 \end{bmatrix}$$

$$r\_jacobi := \left[41, \frac{41}{2}, \frac{41}{4}\right]$$

$$e\_jacobi := \left[19, \frac{19}{2}, \frac{19}{4}\right] \quad (1.5)$$

```
> x4 := P^(-1).(Q.x3+b) ; r4 := b - A.x4 ;
  r_jacobi := [op(r_jacobi), norm(r4)];
  e_jacobi := [op(e_jacobi), norm(x4-xe)];
```

$$x4 := \begin{bmatrix} \frac{11}{8} \\ \frac{19}{4} \\ -\frac{13}{4} \end{bmatrix}$$

$$r4 := \begin{bmatrix} \frac{1}{2} \\ -\frac{41}{4} \\ -\frac{9}{4} \end{bmatrix}$$

$$r\_jacobi := \left[41, \frac{41}{2}, \frac{41}{4}, \frac{41}{4}\right]$$

$$e\_jacobi := \left[19, \frac{19}{2}, \frac{19}{4}, \frac{19}{4}\right] \tag{1.6}$$

```
> x5 := P^(-1).(Q.x4+b) ; r5 := b - A.x5 ;
  r_jacobi := [op(r_jacobi), norm(r5)];
  e_jacobi := [op(e_jacobi), norm(x5-xe)];
```

$$x5 := \begin{bmatrix} \frac{13}{8} \\ -\frac{3}{8} \\ -\frac{35}{8} \end{bmatrix}$$

$$r5 := \begin{bmatrix} \frac{9}{8} \\ -\frac{1}{2} \\ \frac{41}{8} \end{bmatrix}$$

$$r\_jacobi := \left[41, \frac{41}{2}, \frac{41}{4}, \frac{41}{4}, \frac{41}{8}\right]$$

$$e\_jacobi := \left[19, \frac{19}{2}, \frac{19}{4}, \frac{19}{4}, \frac{19}{8}\right] \tag{1.7}$$

```
> x6 := P^(-1).(Q.x5+b) ; r6 := b - A.x6 ;
  r_jacobi := [op(r_jacobi), norm(r6)];
  e_jacobi := [op(e_jacobi), norm(x6-xe)];
```

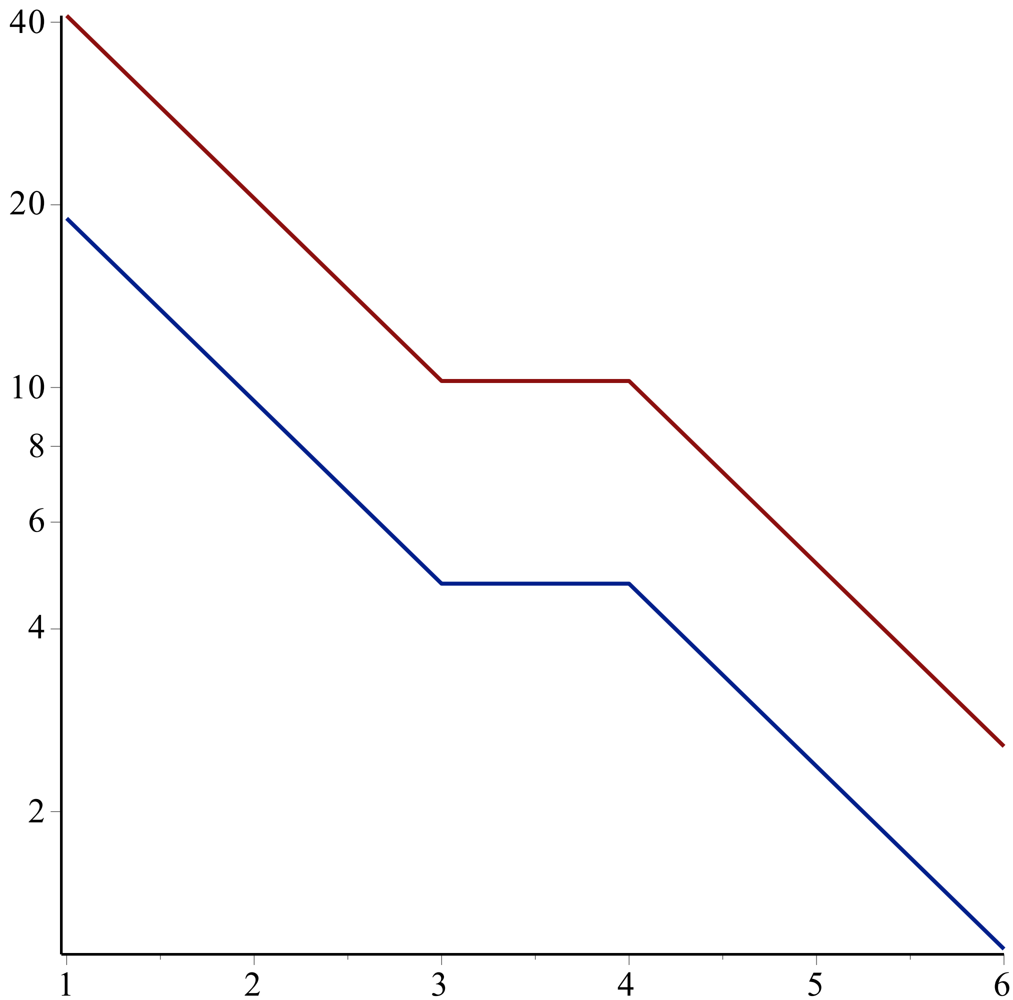
$$x6 := \begin{bmatrix} \frac{35}{16} \\ -\frac{5}{8} \\ -\frac{29}{16} \end{bmatrix}$$

$$r6 := \begin{bmatrix} -\frac{41}{16} \\ -\frac{9}{8} \\ \frac{1}{4} \end{bmatrix}$$

$$r\_jacobi := \left[41, \frac{41}{2}, \frac{41}{4}, \frac{41}{4}, \frac{41}{8}, \frac{41}{16}\right]$$

$$e\_jacobi := \left[19, \frac{19}{2}, \frac{19}{4}, \frac{19}{4}, \frac{19}{8}, \frac{19}{16}\right] \qquad \textbf{(1.8)}$$

```
> logplot([[seq([i,r_jacobi[i]],i=1..6)],[seq([i,e_jacobi[i]],i=1..6)]]);
```



Verifica teorica della convergenza: raggio spettrale di Q.P^(-1)

Modo 1:

```
> Eigenvalues( Q.P^(-1) ) ; evalf(%) ; abs(%) ;
```

$$\begin{bmatrix} -\frac{1}{2}\,2^{1/3} \\ \frac{1}{4}\,2^{1/3}-\frac{1}{4}\,I\sqrt{3}\,2^{1/3} \\ \frac{1}{4}\,2^{1/3}+\frac{1}{4}\,I\sqrt{3}\,2^{1/3} \end{bmatrix}$$

$$\begin{bmatrix} -0.6299605250 \\ 0.3149802625-0.5455618182\,I \\ 0.3149802625+0.5455618182\,I \end{bmatrix}$$

$$\begin{bmatrix} 0.6299605250 \\ 0.6299605251 \\ 0.6299605251 \end{bmatrix} \tag{1.9}$$

```
> Eigenvalues( P^(-1).Q ) ; evalf(%) ;
```

$$\begin{bmatrix} -\frac{1}{2}\,2^{1/3} \\ \frac{1}{4}\,2^{1/3}-\frac{1}{4}\,I\sqrt{3}\,2^{1/3} \\ \frac{1}{4}\,2^{1/3}+\frac{1}{4}\,I\sqrt{3}\,2^{1/3} \end{bmatrix}$$

$$\begin{bmatrix} -0.6299605250 \\ 0.3149802625-0.5455618182\,I \\ 0.3149802625+0.5455618182\,I \end{bmatrix} \tag{1.10}$$

```
> Determinant( Q-lambda*P ) ; solve(%,{lambda}) ; evalf(%) ;
```

$$-8\,\lambda^3-2$$

$$\left\{\lambda=-\frac{1}{2}\,2^{1/3}\right\},\ \left\{\lambda=\frac{1}{4}\,2^{1/3}-\frac{1}{4}\,I\sqrt{3}\,2^{1/3}\right\},\ \left\{\lambda=\frac{1}{4}\,2^{1/3}+\frac{1}{4}\,I\sqrt{3}\,2^{1/3}\right\}$$

$$\{\lambda=-0.6299605250\},\ \{\lambda=0.3149802625-0.5455618182\,I\},\ \{\lambda=0.3149802625+0.5455618182\,I\} \tag{1.11}$$

```
> MM := P^(-1).Q ;
  RS := [op(RS),max(abs(evalf(Eigenvalues(MM))))] ;
  N1 := [op(N1),norm(MM,1)] ;
  NI := [op(NI),norm(MM,infinity)] ;
  N2 := [op(N2),norm(MM,2)] ;
```

$$MM := \begin{bmatrix} 0 & 0 & -\frac{1}{2} \\ -1 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \end{bmatrix}$$

$$RS := [0.6299605251]$$

$$N1 := [1]$$

$$NI := [1]$$

$$N2 := [1] \qquad (1.12)$$

```
> JordanForm(MM) ;
```

$$\begin{bmatrix} -\frac{1}{2}\,2^{1/3} & 0 & 0 \\ 0 & \frac{1}{4}\,2^{1/3} - \frac{1}{4}\,I\sqrt{3}\,2^{1/3} & 0 \\ 0 & 0 & \frac{1}{4}\,2^{1/3} + \frac{1}{4}\,I\sqrt{3}\,2^{1/3} \end{bmatrix} \qquad (1.13)$$

```
> norm(JordanForm(MM),infinity); evalf(%) ;
```

$$\frac{1}{2}\,2^{1/3}$$

$$0.6299605250 \qquad (1.14)$$

Metodo di Gauss-Seidel

Scrivo il metodo nella forma di splitting P-Q

```
>    P := DD+LL ;
  Q := -UU ;
```

$$P := \begin{bmatrix} 2 & 0 & 0 \\ 2 & 2 & 0 \\ 0 & 1 & 2 \end{bmatrix}$$

$$Q := \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad (2.1)$$

Il metodo iterativo è P x[k+1] := Q x[k] + b

10 iterate, andamento del residuo e dell'errore

```
> r_gs := [] ; # vettore con norma residui
  e_gs := [] ; # vettore con norma errore
```

$$r\_gs := [\,]$$

$$e\_gs := [\,] \qquad (2.2)$$

```
> x1 := P^(-1).(Q.x0+b) ; r1 := b - A.x1 :
```

```
r_gs := [op(r_gs), norm(r1)];
e_gs := [op(e_gs), norm(x1-xe)];
```

$$x1 := \begin{bmatrix} -\frac{1}{2} \\ \frac{3}{2} \\ -\frac{11}{4} \end{bmatrix}$$

$$r\_gs := \left[\frac{15}{4}\right]$$

$$e\_gs := \left[\frac{3}{2}\right] \quad \textbf{(2.3)}$$

```
> x2 := P^(-1).(Q.x1+b) ; r2 := b - A.x2 ;
  r_gs := [op(r_gs), norm(r2)];
  e_gs := [op(e_gs), norm(x2-xe)];
```

$$x2 := \begin{bmatrix} \frac{11}{8} \\ -\frac{3}{8} \\ -\frac{29}{16} \end{bmatrix}$$

$$r2 := \begin{bmatrix} -\frac{15}{16} \\ 0 \\ 0 \end{bmatrix}$$

$$r\_gs := \left[\frac{15}{4}, \frac{15}{16}\right]$$

$$e\_gs := \left[\frac{3}{2}, \frac{3}{8}\right] \quad \textbf{(2.4)}$$

```
> x3 := P^(-1).(Q.x2+b) ; r3 := b - A.x3 ;
  r_gs := [op(r_gs), norm(r3)];
  e_gs := [op(e_gs), norm(x3-xe)];
```

$$x3 := \begin{bmatrix} \frac{29}{32} \\ \frac{3}{32} \\ -\frac{131}{64} \end{bmatrix}$$

$$r3 := \begin{bmatrix} \frac{15}{64} \\ 0 \\ 0 \end{bmatrix}$$

$$r\_gs := \left[\frac{15}{4}, \frac{15}{16}, \frac{15}{64} \right]$$

$$e\_gs := \left[\frac{3}{2}, \frac{3}{8}, \frac{3}{32} \right] \quad \textbf{(2.5)}$$

```
> x4 := P^(-1).(Q.x3+b) ; r4 := b - A.x4 ;
  r_gs := [op(r_gs), norm(r4)];
  e_gs := [op(e_gs), norm(x4-xe)];
```

$$x4 := \begin{bmatrix} \frac{131}{128} \\ -\frac{3}{128} \\ -\frac{509}{256} \end{bmatrix}$$

$$r4 := \begin{bmatrix} -\frac{15}{256} \\ 0 \\ 0 \end{bmatrix}$$

$$r\_gs := \left[\frac{15}{4}, \frac{15}{16}, \frac{15}{64}, \frac{15}{256} \right]$$

$$e\_gs := \left[\frac{3}{2}, \frac{3}{8}, \frac{3}{32}, \frac{3}{128} \right] \quad \textbf{(2.6)}$$

```
> x5 := P^(-1).(Q.x4+b) ; r5 := b - A.x5 ;
  r_gs := [op(r_gs), norm(r5)];
  e_gs := [op(e_gs), norm(x5-xe)];
```

$$x5 := \begin{bmatrix} \frac{509}{512} \\ \frac{3}{512} \\ -\frac{2051}{1024} \end{bmatrix}$$

$$r5 := \begin{bmatrix} \frac{15}{1024} \\ 0 \\ 0 \end{bmatrix}$$

$$r\_gs := \left[\frac{15}{4}, \frac{15}{16}, \frac{15}{64}, \frac{15}{256}, \frac{15}{1024}\right]$$

$$e\_gs := \left[\frac{3}{2}, \frac{3}{8}, \frac{3}{32}, \frac{3}{128}, \frac{3}{512}\right] \qquad \textbf{(2.7)}$$

```
> x6 := P^(-1).(Q.x5+b) ; r6 := b - A.x6 ;
  r_gs := [op(r_gs), norm(r6)];
  e_gs := [op(e_gs), norm(x6-xe)];
```

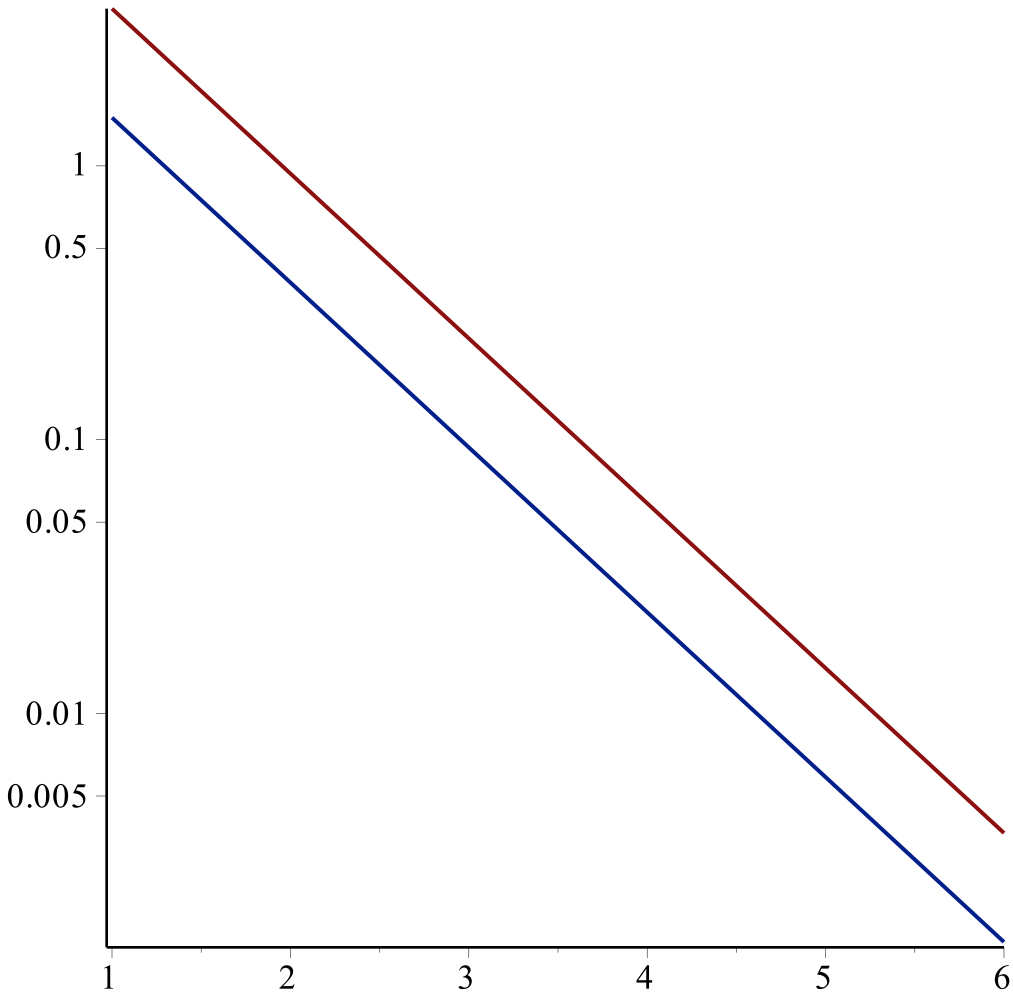
$$x6 := \begin{bmatrix} \frac{2051}{2048} \\ -\frac{3}{2048} \\ -\frac{8189}{4096} \end{bmatrix}$$

$$r6 := \begin{bmatrix} -\frac{15}{4096} \\ 0 \\ 0 \end{bmatrix}$$

$$r\_gs := \left[\frac{15}{4}, \frac{15}{16}, \frac{15}{64}, \frac{15}{256}, \frac{15}{1024}, \frac{15}{4096}\right]$$

$$e\_gs := \left[\frac{3}{2}, \frac{3}{8}, \frac{3}{32}, \frac{3}{128}, \frac{3}{512}, \frac{3}{2048}\right] \qquad \textbf{(2.8)}$$

```
> logplot([[seq([i,r_gs[i]],i=1..6)],[seq([i,e_gs[i]],i=1..6)]]);
```



Verifica teorica della convergenza: raggio spettrale di Q.P^(-1)

Modo 1:

```
> Eigenvalues( Q.P^(-1) ) ; evalf(%) ;
```

$$\begin{bmatrix} 0 \\ 0 \\ -\frac{1}{4} \end{bmatrix}$$

$$\begin{bmatrix} 0. \\ 0. \\ -0.2500000000 \end{bmatrix} \quad (2.9)$$

```
> Eigenvalues( P^(-1).Q ) ; evalf(%) ;
```

$$\begin{bmatrix} 0 \\ 0 \\ -\frac{1}{4} \end{bmatrix}$$

$$\begin{bmatrix} 0. \\ 0. \\ -0.2500000000 \end{bmatrix} \quad (2.10)$$

```
> Determinant( Q-lambda*P ) ; solve(%,{lambda}) ; evalf(%) ;
```

$$-8\lambda^3 - 2\lambda^2$$

$$\{\lambda = 0\}, \{\lambda = 0\}, \left\{\lambda = -\frac{1}{4}\right\}$$

$$\{\lambda = 0.\}, \{\lambda = 0.\}, \{\lambda = -0.2500000000\} \quad (2.11)$$

```
> MM := P^(-1).Q ;
  RS := [op(RS),max(abs(evalf(Eigenvalues(MM))))] ;
  N1 := [op(N1),norm(MM,1)] ;
  NI := [op(NI),norm(MM,infinity)] ;
  N2 := [op(N2),norm(MM,2)] ;
```

$$MM := \begin{bmatrix} 0 & 0 & -\frac{1}{2} \\ 0 & 0 & \frac{1}{2} \\ 0 & 0 & -\frac{1}{4} \end{bmatrix}$$

$$RS := [0.6299605251, 0.2500000000]$$

$$N1 := \left[1, \frac{5}{4}\right]$$

$$NI := \left[1, \frac{1}{2}\right]$$

$$N2 := \left[1, \frac{3}{4}\right] \quad (2.12)$$

Metodo basato su splitting (arbitrario)

Scrivo il metodo nella forma di splitting P-Q

```
> P := <<2,2,0>|<0,1,1>|<1,0,2>> ;
  Q := A-P ;
```

$$P := \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$$

$$Q := \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (3.1)$$

Il metodo iterativo è P x[k+1] := Q x[k] + b

10 iterate, andamento del residuo e dell'errore

```
> r_sa := [] ; # vettore con norma residui
  e_sa := [] ; # vettore con norma errore
```

$$r\_sa := [\,]$$

$$e\_sa := [\,] \qquad (3.2)$$

```
> x1 := P^(-1).(Q.x0+b) ; r1 := b - A.x1 :
  r_sa := [op(r_sa), norm(r1)];
  e_sa := [op(e_sa), norm(x1-xe)];
```

$$x1 := \begin{bmatrix} -\frac{2}{3} \\ -\frac{20}{3} \\ \frac{4}{3} \end{bmatrix}$$

$$r\_sa := \left[\frac{50}{3}\right]$$

$$e\_sa := \left[\frac{20}{3}\right] \qquad (3.3)$$

```
> x2 := P^(-1).(Q.x1+b) ; r2 := b - A.x2 ;
  r_sa := [op(r_sa), norm(r2)];
  e_sa := [op(e_sa), norm(x2-xe)];
```

$$x2 := \begin{bmatrix} -\frac{1}{9} \\ -\frac{40}{9} \\ \frac{2}{9} \end{bmatrix}$$

$$r2 := \begin{bmatrix} 0 \\ \frac{100}{9} \\ 0 \end{bmatrix}$$

$$r\_sa := \left[\frac{50}{3}, \frac{100}{9}\right]$$

$$e\_sa := \left[\frac{20}{3}, \frac{40}{9}\right] \qquad (3.4)$$

```
> x3 := P^(-1).(Q.x2+b) ; r3 := b - A.x3 ;
  r_sa := [op(r_sa), norm(r3)];
  e_sa := [op(e_sa), norm(x3-xe)];
```

$$x3 := \begin{bmatrix} \frac{7}{27} \\ -\frac{80}{27} \\ -\frac{14}{27} \end{bmatrix}$$

$$r3 := \begin{bmatrix} 0 \\ \frac{200}{27} \\ 0 \end{bmatrix}$$

$$r\_sa := \left[\frac{50}{3}, \frac{100}{9}, \frac{200}{27} \right]$$

$$e\_sa := \left[\frac{20}{3}, \frac{40}{9}, \frac{80}{27} \right] \quad \textbf{(3.5)}$$

```
> x4 := P^(-1).(Q.x3+b) ; r4 := b - A.x4 ;
  r_sa := [op(r_sa), norm(r4)];
  e_sa := [op(e_sa), norm(x4-xe)];
```

$$x4 := \begin{bmatrix} \frac{41}{81} \\ -\frac{160}{81} \\ -\frac{82}{81} \end{bmatrix}$$

$$r4 := \begin{bmatrix} 0 \\ \frac{400}{81} \\ 0 \end{bmatrix}$$

$$r\_sa := \left[\frac{50}{3}, \frac{100}{9}, \frac{200}{27}, \frac{400}{81} \right]$$

$$e\_sa := \left[\frac{20}{3}, \frac{40}{9}, \frac{80}{27}, \frac{160}{81} \right] \quad \textbf{(3.6)}$$

```
> x5 := P^(-1).(Q.x4+b) ; r5 := b - A.x5 ;
  r_sa := [op(r_sa), norm(r5)];
  e_sa := [op(e_sa), norm(x5-xe)];
```

$$x5 := \begin{bmatrix} \frac{163}{243} \\ -\frac{320}{243} \\ -\frac{326}{243} \end{bmatrix}$$

$$r5 := \begin{bmatrix} 0 \\ \frac{800}{243} \\ 0 \end{bmatrix}$$

$$r\_sa := \left[\frac{50}{3}, \frac{100}{9}, \frac{200}{27}, \frac{400}{81}, \frac{800}{243}\right]$$

$$e\_sa := \left[\frac{20}{3}, \frac{40}{9}, \frac{80}{27}, \frac{160}{81}, \frac{320}{243}\right] \tag{3.7}$$

```
> x6 := P^(-1).(Q.x5+b) ; r6 := b - A.x6 ;
  r_sa := [op(r_sa), norm(r6)];
  e_sa := [op(e_sa), norm(x6-xe)];
```

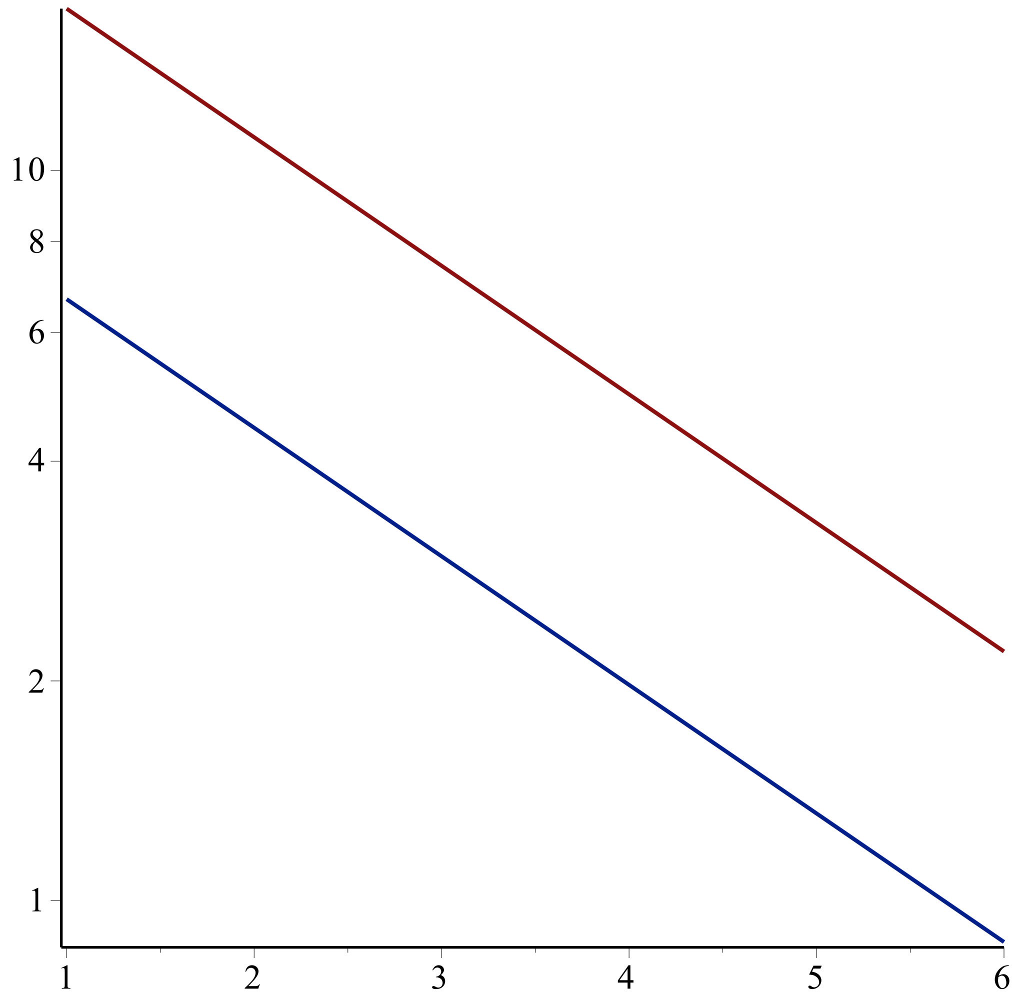
$$x6 := \begin{bmatrix} \frac{569}{729} \\ -\frac{640}{729} \\ -\frac{1138}{729} \end{bmatrix}$$

$$r6 := \begin{bmatrix} 0 \\ \frac{1600}{729} \\ 0 \end{bmatrix}$$

$$r\_sa := \left[\frac{50}{3}, \frac{100}{9}, \frac{200}{27}, \frac{400}{81}, \frac{800}{243}, \frac{1600}{729}\right]$$

$$e\_sa := \left[\frac{20}{3}, \frac{40}{9}, \frac{80}{27}, \frac{160}{81}, \frac{320}{243}, \frac{640}{729}\right] \tag{3.8}$$

```
> logplot([[seq([i,r_sa[i]],i=1..6)],[seq([i,e_sa[i]],i=1..6)]]);
```



Verifica teorica della convergenza: raggio spettrale di Q.P^(-1)

Modo 1:

```
> Eigenvalues( Q.P^(-1) ) ; evalf(%) ; abs(%) ;
```

$$\begin{bmatrix} 0 \\ 0 \\ \frac{2}{3} \end{bmatrix}$$

$$\begin{bmatrix} 0. \\ 0. \\ 0.6666666667 \end{bmatrix}$$

$$\begin{bmatrix} 0. \\ 0. \\ 0.6666666667 \end{bmatrix} \quad (3.9)$$

```
> Eigenvalues( P^(-1).Q ) ; evalf(%) ;
```

$$\begin{bmatrix} 0 \\ 0 \\ \frac{2}{3} \end{bmatrix}$$

$$\begin{bmatrix} 0. \\ 0. \\ 0.6666666667 \end{bmatrix} \quad (3.10)$$

```
> Determinant( Q-lambda*P ) ; solve(%,{lambda}) ; evalf(%) ;
```

$$4\lambda^2 - 6\lambda^3$$

$$\{\lambda = 0\}, \{\lambda = 0\}, \left\{\lambda = \frac{2}{3}\right\}$$

$$\{\lambda = 0.\}, \{\lambda = 0.\}, \{\lambda = 0.6666666667\} \quad (3.11)$$

```
> MM := P^(-1).Q ;
  RS := [op(RS),max(abs(evalf(Eigenvalues(MM))))] ;
  N1 := [op(N1),norm(MM,1)] ;
  NI := [op(NI),norm(MM,infinity)] ;
  N2 := [op(N2),norm(MM,2)] ;
```

$$MM := \begin{bmatrix} 0 & \frac{1}{6} & 0 \\ 0 & \frac{2}{3} & 0 \\ 0 & -\frac{1}{3} & 0 \end{bmatrix}$$

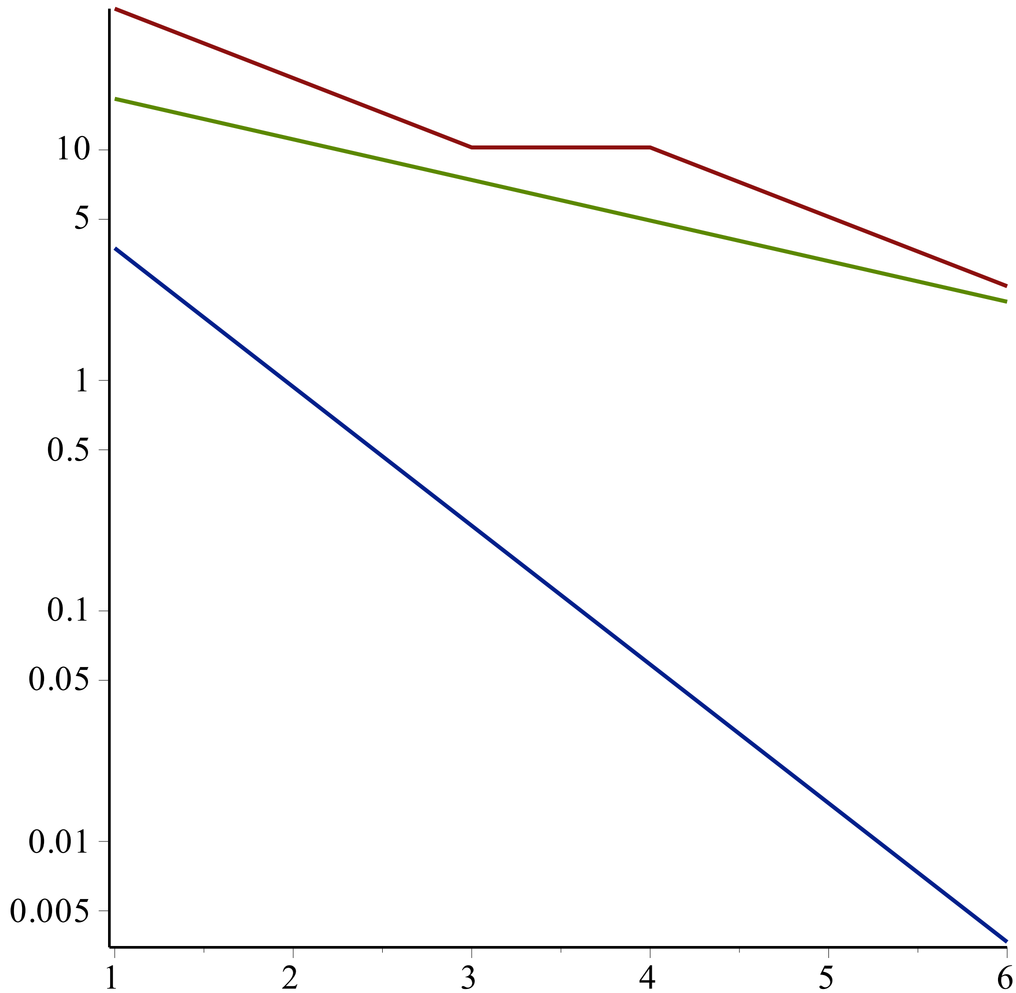
$$RS := [0.6299605251, 0.2500000000, 0.6666666667]$$

$$N1 := \left[1, \frac{5}{4}, \frac{7}{6}\right]$$

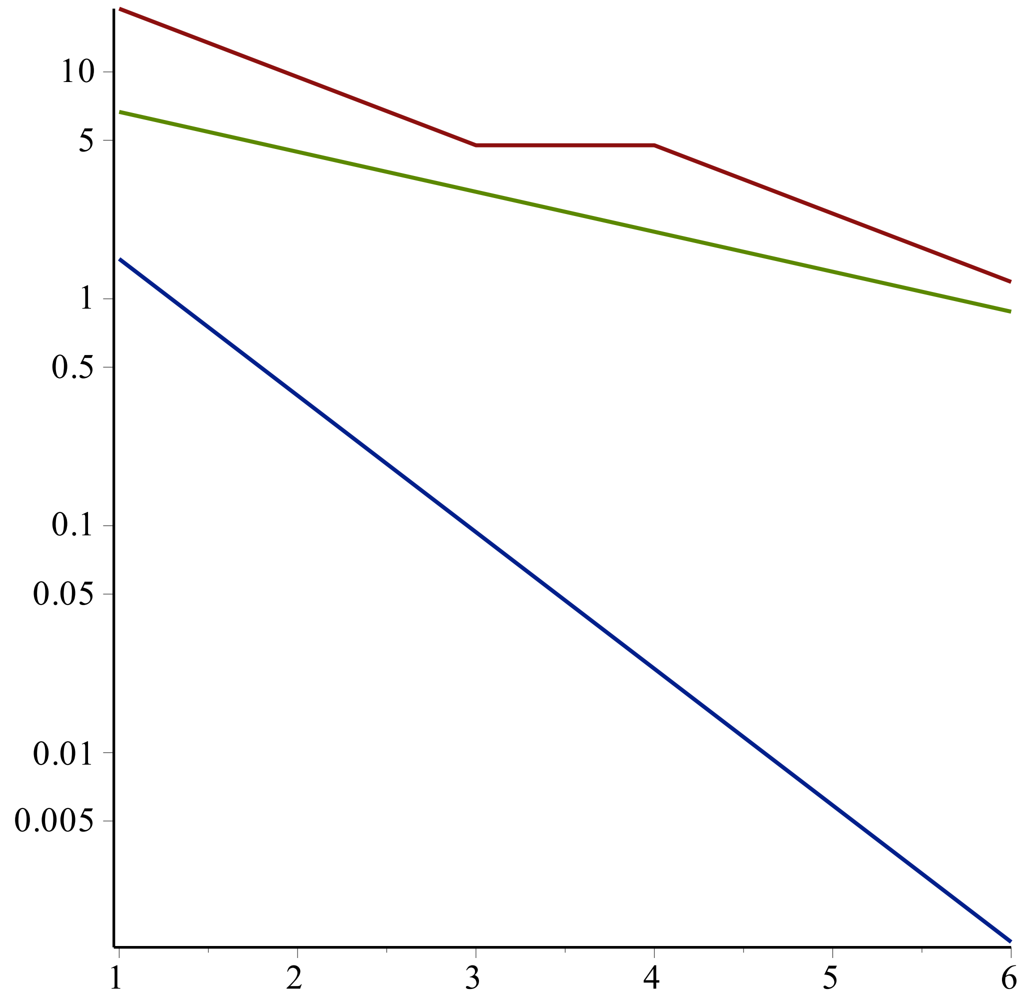
$$NI := \left[1, \frac{1}{2}, \frac{2}{3}\right]$$

$$N2 := \left[1, \frac{3}{4}, \frac{1}{6}\sqrt{21}\right] \quad (3.12)$$

```
> logplot([[seq([i,r_jacobi[i]],i=1..6)],
           [seq([i,r_gs[i]],i=1..6)],
           [seq([i,r_sa[i]],i=1..6)]]);
```




```
> logplot([[seq([i,e_jacobi[i]],i=1..6)],
          [seq([i,e_gs[i]],i=1..6)],
          [seq([i,e_sa[i]],i=1..6)]]);
```




```
> RS ;
```

[0.6299605251, 0.2500000000, 0.6666666667] **(5)**

```
> evalf(NI);
  evalf(N2);
  evalf(N1);
```

[1., 0.5000000000, 0.6666666667]

[1., 0.7500000000, 0.7637626160]

[1., 1.250000000, 1.166666667] **(6)**