Derivazione del metodo di Milne

```
> restart:
> with(CurveFitting);
```

$$[ArrayInterpolation, BSpline, BSplineCurve, Interactive, LeastSquares, PolynomialInterpolation, RationalInterpolation, Spline, ThieleInterpolation] \quad (1)$$

Passo esplicito

```
> XY := [seq( [x[k]-j*h,f[k-j]],j=0..2)] ;
```

$$XY := [[x_k, f_k], [-h + x_k, f_{k-1}], [-2h + x_k, f_{k-2}]] \quad (2)$$

```
> Pint := PolynomialInterpolation( XY, z) ;
```

$$Pint := \frac{1}{2} \frac{(f_{k-2} - 2 f_{k-1} + f_k) z^2}{h^2} + \frac{1}{2} \frac{(3 h f_k + h f_{k-2} - 4 h f_{k-1} - 2 f_k x_k - 2 f_{k-2} x_k + 4 f_{k-1} x_k) z}{h^2} + \frac{1}{2} \frac{2 h^2 f_k - 3 h f_k x_k - h f_{k-2} x_k + 4 h f_{k-1} x_k + f_k x_k^2 + f_{k-2} x_k^2 - 2 f_{k-1} x_k^2}{h^2} \quad (3)$$

```
> IntP := simplify(int( Pint, z=x[k]-3*h..x[k]+h)) ; # integro tra x
  [k-3]..x[k+1]
```

$$IntP := \frac{4}{3} h (2 f_k + 2 f_{k-2} - f_{k-1}) \quad (4)$$

Predittore esplicito

```
> Predictor := ytilde[k+1]=y[k-3]+collect(IntP,[h,f]) ;
```

$$Predictor := ytilde_{k+1} = y_{k-3} + \left(\frac{8}{3} f_k + \frac{8}{3} f_{k-2} - \frac{4}{3} f_{k-1}\right) h \quad (5)$$

Passo implicito

```
> XY := [seq( [x[k]-j*h,f[k-j]],j=-1..1)] ;
```

$$XY := [[x_k + h, f_{k+1}], [x_k, f_k], [-h + x_k, f_{k-1}]] \quad (6)$$

```
> Pint := PolynomialInterpolation( XY, z) ;
```

$$Pint := \frac{1}{2} \frac{(f_{k-1} - 2 f_k + f_{k+1}) z^2}{h^2} + \frac{1}{2} \frac{(-h f_{k-1} + h f_{k+1} + 4 f_k x_k - 2 f_{k-1} x_k - 2 f_{k+1} x_k) z}{h^2} + \frac{1}{2} \frac{2 h^2 f_k + h f_{k-1} x_k - h f_{k+1} x_k - 2 f_k x_k^2 + f_{k-1} x_k^2 + f_{k+1} x_k^2}{h^2} \quad (7)$$

```
> IntP := simplify(int( Pint, z=x[k]-h..x[k]+h)) ; # integrro tra x
  [k-1]..x[k+1]
```

$$IntP := \frac{1}{3} h (4 f_k + f_{k-1} + f_{k+1}) \quad (8)$$

Correttore (implicito)

```
> Corrector := y[k+1]=y[k-1]+collect(IntP,[h,f]) ;
```

$$Corrector := y_{k+1} = y_{k-1} + \left(\frac{4}{3} f_k + \frac{1}{3} f_{k-1} + \frac{1}{3} f_{k+1}\right) h \qquad \textbf{(9)}$$