

Esempio di Eliminazione di Gauss (senza pivoting) (17 febbraio 2014)

> **with(LinearAlgebra) :**

Matrice dei coefficienti del sistema

> **A := <<1,2,3,4>|<4,3,2,1>|<0,1,-1,0>|<1,0,0,0>> ;**

$$A := \begin{bmatrix} 1 & 4 & 0 & 1 \\ 2 & 3 & 1 & 0 \\ 3 & 2 & -1 & 0 \\ 4 & 1 & 0 & 0 \end{bmatrix}$$

(1)

Soluzione

> **x := <1,2,3,4> ;**

$$x := \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

(2)

Termine noto

> **b := A.x ;**

$$b := \begin{bmatrix} 13 \\ 11 \\ 4 \\ 6 \end{bmatrix}$$

(3)

Impacchetto il sistema in una unica matrice.

> **T := ArrayTools[Concatenate](2,A,b) ;**

$$T := \begin{bmatrix} 1 & 4 & 0 & 1 & 13 \\ 2 & 3 & 1 & 0 & 11 \\ 3 & 2 & -1 & 0 & 4 \\ 4 & 1 & 0 & 0 & 6 \end{bmatrix}$$

(4)

Le incognite sono x1,x2,x3,x4.

Passo1 elimino x1 dalle equazioni 2,3,4

> **T1 := <<0,0,0,0>|<0,0,0,0>|<0,0,0,0>|<0,0,0,0>|<0,0,0,0>> :**

> **T1[1,1..5] := T[1,1..5] ;**

T1[2,1..5] := T[2,1..5]-2*T[1,1..5] ; # (2) -> (2)-2(1)

T1[3,1..5] := T[3,1..5]-3*T[1,1..5] ; # (3) -> (3)-3(1)

T1[4,1..5] := T[4,1..5]-4*T[1,1..5] ; # (4) -> (4)-4(1)

$$T1_{1,1..5} := \begin{bmatrix} 1 & 4 & 0 & 1 & 13 \end{bmatrix}$$

$$T1_{2,1..5} := \begin{bmatrix} 0 & -5 & 1 & -2 & -15 \end{bmatrix}$$

$$\begin{aligned}
 T1_{3,1..5} &:= \begin{bmatrix} 0 & -10 & -1 & -3 & -35 \end{bmatrix} \\
 T1_{4,1..5} &:= \begin{bmatrix} 0 & -15 & 0 & -4 & -46 \end{bmatrix}
 \end{aligned}
 \tag{5}$$

Matrice dopo la prima eliminazione

> **T1** ;

$$\begin{bmatrix} 1 & 4 & 0 & 1 & 13 \\ 0 & -5 & 1 & -2 & -15 \\ 0 & -10 & -1 & -3 & -35 \\ 0 & -15 & 0 & -4 & -46 \end{bmatrix}
 \tag{6}$$

Passo1 elimino x2 dalle equazioni 3,4

> **T2** := <<0,0,0,0>|<0,0,0,0>|<0,0,0,0>|<0,0,0,0>|<0,0,0,0>> :

> **T2**[1..2,1..5] := **T1**[1..2,1..5] ;

T2[3,1..5] := **T1**[3,1..5]-2***T1**[2,1..5] ; # (3) -> (3)-2(1)

T2[4,1..5] := **T1**[4,1..5]-3***T1**[2,1..5] ; # (4) -> (4)-3(1)

$$T2_{1..2,1..5} := \begin{bmatrix} 1 & 4 & 0 & 1 & 13 \\ 0 & -5 & 1 & -2 & -15 \end{bmatrix}$$

$$T2_{3,1..5} := \begin{bmatrix} 0 & 0 & -3 & 1 & -5 \end{bmatrix}$$

$$T2_{4,1..5} := \begin{bmatrix} 0 & 0 & -3 & 2 & -1 \end{bmatrix}
 \tag{7}$$

Matrice dopo la seconda eliminazione

> **T2** ;

$$\begin{bmatrix} 1 & 4 & 0 & 1 & 13 \\ 0 & -5 & 1 & -2 & -15 \\ 0 & 0 & -3 & 1 & -5 \\ 0 & 0 & -3 & 2 & -1 \end{bmatrix}
 \tag{8}$$

Passo2 elimino x3 dalla equazione 4

> **T3** := <<0,0,0,0>|<0,0,0,0>|<0,0,0,0>|<0,0,0,0>|<0,0,0,0>> :

> **T3**[1..3,1..5] := **T2**[1..3,1..5] ;

T3[4,1..5] := **T2**[4,1..5]-**T2**[3,1..5] ; # (4) -> (4)-(3)

$$T3_{1..3,1..5} := \begin{bmatrix} 1 & 4 & 0 & 1 & 13 \\ 0 & -5 & 1 & -2 & -15 \\ 0 & 0 & -3 & 1 & -5 \end{bmatrix}$$

$$T3_{4,1..5} := \begin{bmatrix} 0 & 0 & 0 & 1 & 4 \end{bmatrix}
 \tag{9}$$

Matrice dopo la seconda eliminazione

> **T3** ;

(10)

$$\begin{bmatrix} 1 & 4 & 0 & 1 & 13 \\ 0 & -5 & 1 & -2 & -15 \\ 0 & 0 & -3 & 1 & -5 \\ 0 & 0 & 0 & 1 & 4 \end{bmatrix} \quad (10)$$

Matrice dopo l'eliminazione

```
> AA := T3[1..4,1..4] ;
```

$$AA := \begin{bmatrix} 1 & 4 & 0 & 1 \\ 0 & -5 & 1 & -2 \\ 0 & 0 & -3 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (11)$$

Termine noto dopo l'eliminazione

```
> bb := T3[1..4,5] ;
```

$$bb := \begin{bmatrix} 13 \\ -15 \\ -5 \\ 4 \end{bmatrix} \quad (12)$$

```
> AA^(-1).bb ;
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$$\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \quad (13)$$