Esempio uso formule di quadratura

Funzione da integrare in maniera approssimata

```
> f := x -> x*cos(x)*exp(x)/(1+x^2) ;
```

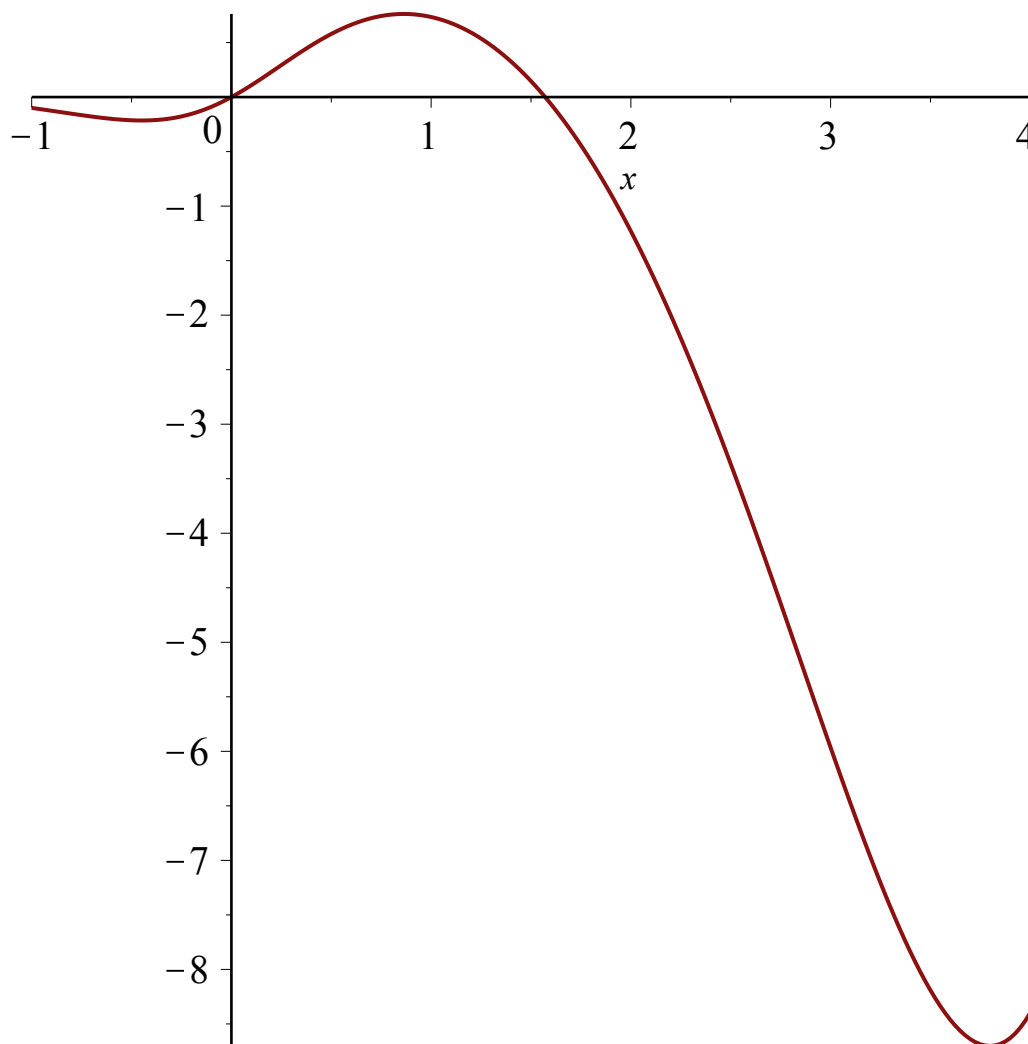$$f := x \rightarrow \frac{x \cos(x)\, e^x}{1 + x^2} \tag{1}$$

Intervallo di integrazione

```
> a,b := -1,4 ;
```

$$a, b := -1, 4 \tag{2}$$

```
> plot( f(x),x=a..b ) ;
```


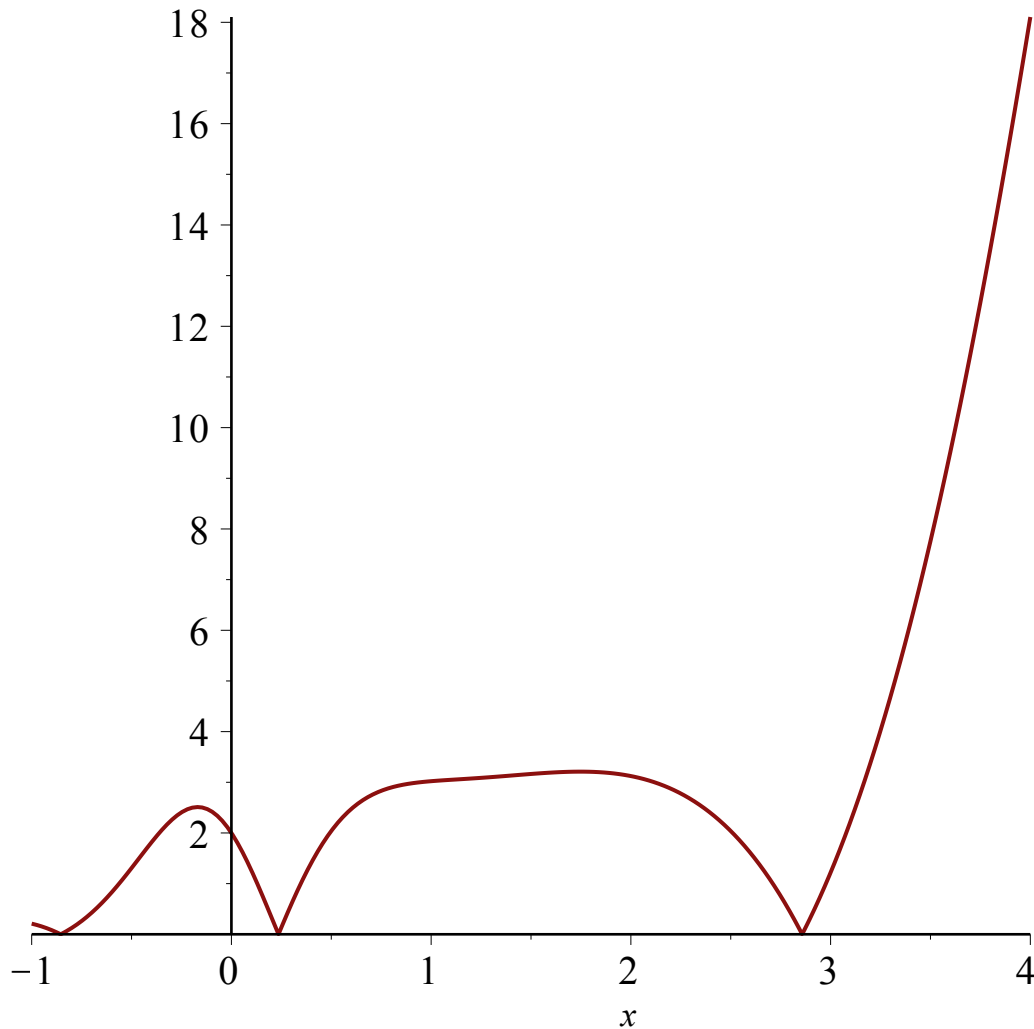Stima dell'errore con metodo dei trapezi, serve la derivata seconda

```
> ddf := simplify(D(D(f))(x)) ;
```

$$ddf := -\frac{1}{(x^2 + 1)^3}\big(2\, e^x \big(\sin(x)\, x^5 - \sin(x)\, x^4 + \cos(x)\, x^4 + 2 \sin(x)\, x^3 - \cos(x)\, x^3 \tag{3}$$
$$+ x \sin(x) + 3\, x \cos(x) + \sin(x) - \cos(x)\big)\big)$$

Stimare massimo modulo derivata seconda nell'intervallo [a,b] = [-1,4]

```
> plot( abs(ddf(x)), x=a..b ) ;
```



```
> stima_ddf := (2*exp(x)*(abs(sin(x))*abs(x^5)+abs(sin(x))*abs(x^4)+
  abs(cos(x))*abs(x^4)+2*abs(sin(x))*abs(x^3)+abs(cos(x))*abs(x^3)+
  abs(x)*abs(sin(x))+3*abs(x)*abs(cos(x))+abs(sin(x))+abs(cos(x))))/
  (x^2+1)^3 ;
```

$$stima\_ddf := \frac{1}{\left(x^2+1\right)^3}\left(2\,e^x\left(|\sin(x)|\,|x|^5 + |\sin(x)|\,|x|^4 + |\cos(x)|\,|x|^4 + 2\,|\sin(x)|\,|x|^3 \right.\right. \tag{4}$$
$$\left.\left. + |\cos(x)|\,|x|^3 + |x|\,|\sin(x)| + 3\,|x|\,|\cos(x)| + |\sin(x)| + |\cos(x)|\right)\right)$$

max |f(x)/g(x)| <= max |f(x)|/min |g(x)|

```
> stima_ddf := (2*exp(x)*(abs(sin(x))*abs(x^5)+abs(sin(x))*abs(x^4)+
  abs(cos(x))*abs(x^4)+2*abs(sin(x))*abs(x^3)+abs(cos(x))*abs(x^3)+
  abs(x)*abs(sin(x))+3*abs(x)*abs(cos(x))+abs(sin(x))+abs(cos(x))));
```

$$stima\_ddf := 2\,e^x\left(|\sin(x)|\,|x|^5 + |\sin(x)|\,|x|^4 + |\cos(x)|\,|x|^4 + 2\,|\sin(x)|\,|x|^3 + |\cos(x)|\,|x|^3 \right. \tag{5}$$
$$\left. + |x|\,|\sin(x)| + 3\,|x|\,|\cos(x)| + |\sin(x)| + |\cos(x)|\right)$$

max |f(x)||g(x)| <= max |f(x)| max |g(x)| --> exp(b) e |sin(x)| <= 1, |cos(x)| <= 1

```
> stima_ddf := (2*exp(b)*(abs(x^5)+abs(x^4)+abs(x^4)+2*abs(x^3)+abs
  (x^3)+abs(x)+3*abs(x)+2));
```

$$stima\_ddf := 2\,e^4\left(|x|^5 + 2\,|x|^4 + 3\,|x|^3 + 4\,|x| + 2\right) \tag{6}$$

|x| <= b in questo caso
```
> stima_ddf := (2*exp(b)*(b^5+b^4+b^4+2*b^3+b^3+b+3*b+2));
  stima_ddf := evalf(%) ;
```

$$stima\_ddf := 3492\,e^4$$

$$stima\_ddf := 1.906567399\,10^5 \tag{7}$$

Errore massimo ammesso 10^(-6), con trapezi E = -(b-a)*h^2/12 f''(eta) -->
|E| <= (b-a)h^2/12 stima_ddf <= 10^(-6)
```
> #stima_ddf := 20 ;
> EQ := (b-a)*h^2/12*stima_ddf = 1e-6 ;
```

$$EQ := 79440.30829\,h^2 = 0.000001 \tag{8}$$

Calcolo intervallo h che soddisfa l'errore
```
> SOL := solve( EQ, {h} ) ;
```

$$SOL := \{h = 0.000003547966737\},\ \{h = -0.000003547966737\} \tag{9}$$

Avendo l'intervallo calcolo n = (b-a)/h
```
> subs(SOL[1],(b-a)/h) ;
```

$$1.409257857\,10^6 \tag{10}$$

Stesso calcolo usando la regola di Simpson

Stima dell'errore con metodo dei trapezi, serve la derivata seconda
```
> ddddf := simplify((D@@4)(f)(x)) ;
```

$$ddddf := \frac{1}{\left(x^2+1\right)^5}\big(4\,e^x\,\big(-\cos(x)\,x^9 + 2\sin(x)\,x^8 + 2\cos(x)\,x^8 - 6\sin(x)\,x^7 \tag{11}$$
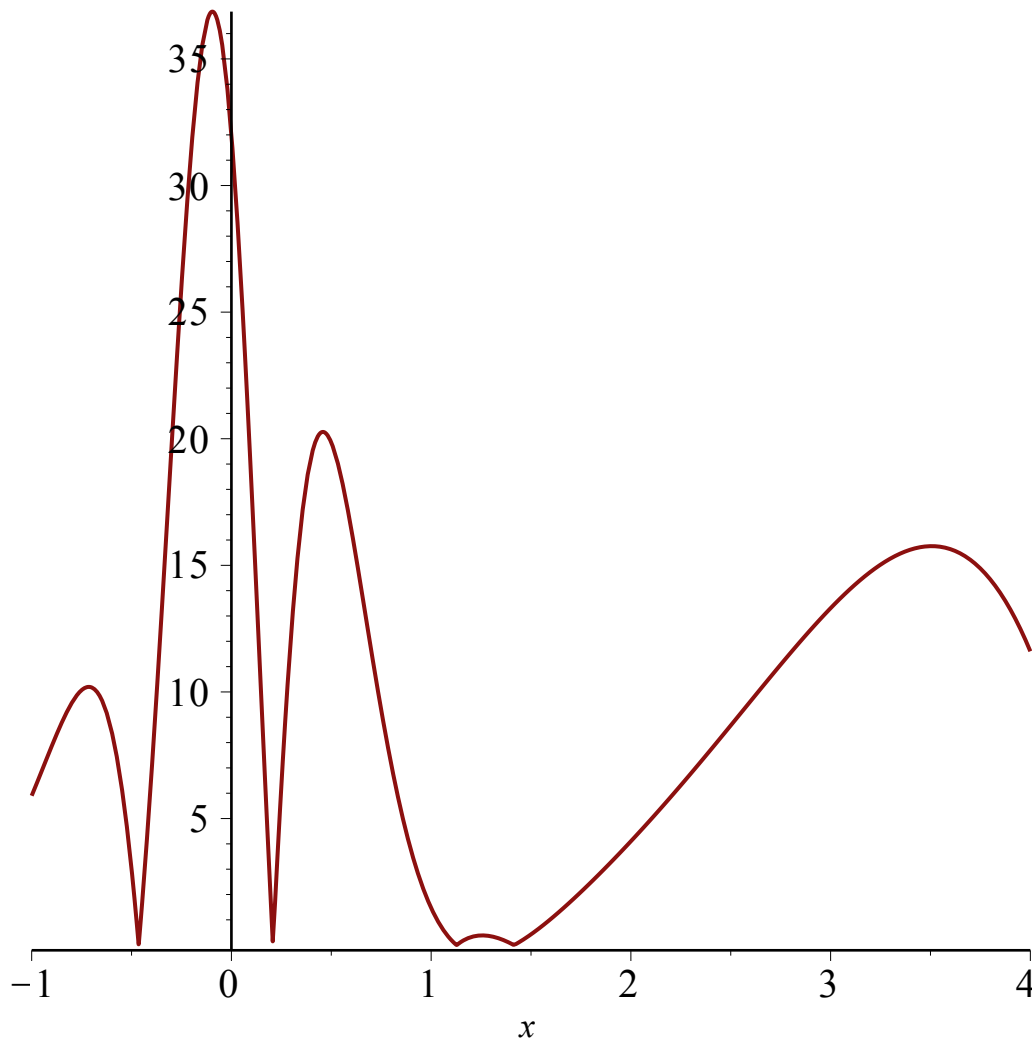
$$-4\cos(x)\,x^7 + 10\sin(x)\,x^6 - 2\cos(x)\,x^6 + 6\sin(x)\,x^5 - 30\sin(x)\,x^4 + 30\cos(x)\,x^4$$

$$+30\sin(x)\,x^3 - 64\cos(x)\,x^3 - 34\,x^2\sin(x) + 26\,x^2\cos(x) + 18\,x\sin(x)$$

$$+29\,x\cos(x) + 4\sin(x) - 8\cos(x)\big)\big)$$

Stimare massimo modulo derivata seconda nell'intervallo [a,b] = [-1,4]
```
> plot( abs(ddddf(x)), x=a..b ) ;
```

Uso diseguagliaza triangolare e max|sin(x)|=max|cos(x)|=1

```
> stima_ddddf := (4*exp(x)*(abs(x)^9+2*abs(x)^8+2*abs(x)^8+6*abs(x)
  ^7*abs(x)^7+10*abs(x)^6+2*abs(x)^6+6*abs(x)^5+30*abs(x)^4+30*abs(x)
  ^4+30*abs(x)^3+64*abs(x)^3+34*abs(x)^2+26*abs(x)^2+18*abs(x)+29*abs
  (x)+4+8))/(x^2+1)^3 ;
```

$stima\_ddddf :=$ **(12)**

$$\frac{4\,e^x\left(|x|^9 + 4\,|x|^8 + 6\,|x|^{14} + 12\,|x|^6 + 6\,|x|^5 + 60\,|x|^4 + 94\,|x|^3 + 60\,|x|^2 + 47\,|x| + 12\right)}{\left(x^2 + 1\right)^3}$$

max |f(x)/g(x)| <= max |f(x)|/min |g(x)|

```
> stima_ddddf := (4*exp(x)*(abs(x)^9+2*abs(x)^8+2*abs(x)^8+6*abs(x)
  ^7*abs(x)^7+10*abs(x)^6+2*abs(x)^6+6*abs(x)^5+30*abs(x)^4+30*abs(x)
  ^4+30*abs(x)^3+64*abs(x)^3+34*abs(x)^2+26*abs(x)^2+18*abs(x)+29*abs
  (x)+4+8));
```

$stima\_ddddf := 4\,e^x\left(|x|^9 + 4\,|x|^8 + 6\,|x|^{14} + 12\,|x|^6 + 6\,|x|^5 + 60\,|x|^4 + 94\,|x|^3 + 60\,|x|^2 \right.$ **(13)**

$\left. + 47\,|x| + 12\right)$

max |f(x)‖g(x)| <= max |f(x)| max |g(x)| --> exp(b)

```
> stima_ddddf := (4*exp(b)*(abs(b)^9+2*abs(b)^8+2*abs(b)^8+6*abs(b)
  ^7*abs(b)^7+10*abs(b)^6+2*abs(b)^6+6*abs(b)^5+30*abs(b)^4+30*abs(b)
  ^4+30*abs(b)^3+64*abs(b)^3+34*abs(b)^2+26*abs(b)^2+18*abs(b)+29*abs
```

```
(b)+4+8));
stima_ddddf := evalf(%) ;
```

$$stima\_ddddf := 6444859424\ e^4$$

$$stima\_ddddf := 3.518774018\ 10^{11} \tag{14}$$

Errore massimo ammesso 10^(-6), con trapezi E = -(b-a)*h^4/180 f''''(eta) -->
|E| <= (b-a)h^4/180 stima_ddf <= 10^(-6)

```
> #stima_ddddf := 35 ;
> EQ := (b-a)*h^4/180*stima_ddddf = 1e-6 ;
```

$$EQ := 9.774372272\ 10^9\ h^4 = 0.000001 \tag{15}$$

Calcolo intervallo h che soddisfa l'errore

```
> SOL := solve( EQ, {h} ) ;
```

$SOL := \{h = 0.0001005721608\}, \{h = 0.0001005721608\ I\}, \{h = -0.0001005721608\}, \{h = -0.0001005721608\ I\}$ $\tag{16}$

Avendo l'intervallo calcolo n = (b-a)/h

```
> subs(SOL[1],(b-a)/h) ;
```

$$49715.54713 \tag{17}$$