Esempio metodo basato su Taylor

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> restart:
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y' = f(x,y)

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> TAYLOR := y(x+h) = y(x) + diff(y(x),x) * h +
                           diff(y(x),x,x) * h^2/2 +
                           diff(y(x),x,x,x) * h^3/3! +
                           diff(y(x),x,x,x,x) * h^4/4! + E ;
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$$TAYLOR := y(x+h) = y(x) + \left(\frac{d}{dx}\, y(x)\right) h + \frac{1}{2}\left(\frac{d^2}{dx^2}\, y(x)\right) h^2 + \frac{1}{6}\left(\frac{d^3}{dx^3}\, y(x)\right) h^3 + \frac{1}{24}\left(\frac{d^4}{dx^4}\, y(x)\right) h^4 + E \quad (1)$$

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> EQD1 := diff(y(x),x) = f(x,y(x)) ;
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$$EQD1 := \frac{d}{dx}\, y(x) = f(x, y(x)) \quad (2)$$

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> EQD2 := diff(y(x),x,x) = diff(f(x,y(x)),x) ;
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$$EQD2 := \frac{d^2}{dx^2}\, y(x) = D_1(f)(x, y(x)) + D_2(f)(x, y(x)) \left(\frac{d}{dx}\, y(x)\right) \quad (3)$$

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> EQD2 := lhs(EQD2) = subs(EQD1,rhs(EQD2)) ;
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$$EQD2 := \frac{d^2}{dx^2}\, y(x) = D_1(f)(x, y(x)) + D_2(f)(x, y(x))\, f(x, y(x)) \quad (4)$$

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> EQD3 := diff(y(x),x,x,x) = diff(f(x,y(x)),x,x) ;
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$$EQD3 := \frac{d^3}{dx^3}\, y(x) = D_{1,1}(f)(x, y(x)) + D_{1,2}(f)(x, y(x)) \left(\frac{d}{dx}\, y(x)\right) + \left(D_{1,2}(f)(x, y(x)) + D_{2,2}(f)(x, y(x)) \left(\frac{d}{dx}\, y(x)\right)\right) \left(\frac{d}{dx}\, y(x)\right) + D_2(f)(x, y(x)) \left(\frac{d^2}{dx^2}\, y(x)\right) \quad (5)$$

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> EQD3 := lhs(EQD3) = subs(EQD1,expand(subs(EQD1,rhs(EQD3)))) ;
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$$EQD3 := \frac{d^3}{dx^3}\, y(x) = D_{1,1}(f)(x, y(x)) + 2\, D_{1,2}(f)(x, y(x))\, f(x, y(x)) + D_{2,2}(f)(x, y(x))\, f(x, y(x))^2 + D_2(f)(x, y(x))\, D_1(f)(x, y(x)) + D_2(f)(x, y(x))^2\, f(x, y(x)) \quad (6)$$

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> EQD4 := diff(y(x),x,x,x,x) = diff(f(x,y(x)),x,x,x) ;
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$$EQD4 := \frac{d^4}{dx^4}\, y(x) = D_{1,1,1}(f)(x, y(x)) + D_{1,1,2}(f)(x, y(x)) \left(\frac{d}{dx}\, y(x)\right) + \left(D_{1,1,2}(f)(x, y(x)) + D_{1,2,2}(f)(x, y(x)) \left(\frac{d}{dx}\, y(x)\right)\right) \left(\frac{d}{dx}\, y(x)\right) + D_{1,2}(f)(x, y(x)) \left(\frac{d^2}{dx^2}\, y(x)\right) + \Big(D_{1,1,2}(f)(x, y(x)) + D_{1,2,2}(f)(x, \quad (7)$$

$y(x))\left(\frac{d}{dx}y(x)\right)+\left(D_{1,2,2}(f)(x,y(x))+D_{2,2,2}(f)(x,\right.$

$\left.y(x))\left(\frac{d}{dx}y(x)\right)\right)\left(\frac{d}{dx}y(x)\right)+D_{2,2}(f)(x,y(x))\left(\frac{d^2}{dx^2}y(x)\right)\Bigg)\left(\frac{d}{dx}y(x)\right)$

$+2\left(D_{1,2}(f)(x,y(x))+D_{2,2}(f)(x,y(x))\left(\frac{d}{dx}y(x)\right)\right)\left(\frac{d^2}{dx^2}y(x)\right)+D_2(f)(x,$

$y(x))\left(\frac{d^3}{dx^3}y(x)\right)$

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> EQD4 := lhs(EQD4) = subs(EQD1,expand(subs(EQD1,expand(subs(EQD1,rhs
  (EQD4)))))) ;
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$$EQD4:=\frac{d^4}{dx^4}y(x)=D_{1,1,1}(f)(x,y(x))+3D_{1,1,2}(f)(x,y(x))f(x,y(x))$$
$$+3D_{1,2,2}(f)(x,y(x))f(x,y(x))^2+3D_{1,2}(f)(x,y(x))D_1(f)(x,y(x))$$
$$+5D_{1,2}(f)(x,y(x))D_2(f)(x,y(x))f(x,y(x))+D_{2,2,2}(f)(x,y(x))f(x,y(x))^3$$
$$+3f(x,y(x))D_{2,2}(f)(x,y(x))D_1(f)(x,y(x))+4f(x,y(x))^2D_{2,2}(f)(x,$$
$$y(x))D_2(f)(x,y(x))+D_2(f)(x,y(x))D_{1,1}(f)(x,y(x))+D_2(f)(x,$$
$$y(x))^2D_1(f)(x,y(x))+D_2(f)(x,y(x))^3f(x,y(x)) \quad \textbf{(8)}$$

Derivate della soluzione esatta in fuzione delle derivate parziali di f(x,y)

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> EQD1 ;
  EQD2 ;
  EQD3 ;
  EQD4 ;
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$$\frac{d}{dx}y(x)=f(x,y(x))$$

$$\frac{d^2}{dx^2}y(x)=D_1(f)(x,y(x))+D_2(f)(x,y(x))f(x,y(x))$$

$$\frac{d^3}{dx^3}y(x)=D_{1,1}(f)(x,y(x))+2D_{1,2}(f)(x,y(x))f(x,y(x))+D_{2,2}(f)(x,y(x))f(x,$$
$$y(x))^2+D_2(f)(x,y(x))D_1(f)(x,y(x))+D_2(f)(x,y(x))^2f(x,y(x))$$

$$\frac{d^4}{dx^4}y(x)=D_{1,1,1}(f)(x,y(x))+3D_{1,1,2}(f)(x,y(x))f(x,y(x))+3D_{1,2,2}(f)(x,$$
$$y(x))f(x,y(x))^2+3D_{1,2}(f)(x,y(x))D_1(f)(x,y(x))+5D_{1,2}(f)(x,$$
$$y(x))D_2(f)(x,y(x))f(x,y(x))+D_{2,2,2}(f)(x,y(x))f(x,y(x))^3+3f(x,$$
$$y(x))D_{2,2}(f)(x,y(x))D_1(f)(x,y(x))+4f(x,y(x))^2D_{2,2}(f)(x,y(x))D_2(f)(x,$$
$$y(x))+D_2(f)(x,y(x))D_{1,1}(f)(x,y(x))+D_2(f)(x,y(x))^2D_1(f)(x,y(x)) \quad \textbf{(9)}$$

$$+ D_2(f)(x, y(x))^3 f(x, y(x))$$

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> EQD2;
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$$\frac{d^2}{dx^2} y(x) = D_1(f)(x, y(x)) + D_2(f)(x, y(x)) f(x, y(x)) \quad (10)$$

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> TAYLOR ;
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$$y(x+h) = y(x) + \left(\frac{d}{dx} y(x)\right) h + \frac{1}{2}\left(\frac{d^2}{dx^2} y(x)\right) h^2 + \frac{1}{6}\left(\frac{d^3}{dx^3} y(x)\right) h^3 + \frac{1}{24}\left(\frac{d^4}{dx^4} y(x)\right) h^4 + E \quad (11)$$

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> NEW_TAYLOR := collect( subs(EQD1,subs(EQD2,subs(EQD3,subs(EQD4,
  TAYLOR)))),[h]);
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$$NEW\_TAYLOR := y(x+h) = \left(\frac{1}{24} D_{2,2,2}(f)(x, y(x)) f(x, y(x))^3 + \frac{1}{6} f(x, y(x))^2 D_{2,2}(f)(x, y(x)) D_2(f)(x, y(x)) + \frac{1}{24} D_2(f)(x, y(x))^3 f(x, y(x)) + \frac{1}{8} D_{1,2,2}(f)(x, y(x)) f(x, y(x))^2 + \frac{1}{8} f(x, y(x)) D_{2,2}(f)(x, y(x)) D_1(f)(x, y(x)) + \frac{5}{24} D_{1,2}(f)(x, y(x)) D_2(f)(x, y(x)) f(x, y(x)) + \frac{1}{24} D_2(f)(x, y(x))^2 D_1(f)(x, y(x)) + \frac{1}{8} D_{1,1,2}(f)(x, y(x)) f(x, y(x)) + \frac{1}{8} D_{1,2}(f)(x, y(x)) D_1(f)(x, y(x)) + \frac{1}{24} D_2(f)(x, y(x)) D_{1,1}(f)(x, y(x)) + \frac{1}{24} D_{1,1,1}(f)(x, y(x))\right) h^4 + \left(\frac{1}{6} D_{2,2}(f)(x, y(x)) f(x, y(x))^2 + \frac{1}{6} D_2(f)(x, y(x))^2 f(x, y(x)) + \frac{1}{3} D_{1,2}(f)(x, y(x)) f(x, y(x)) + \frac{1}{6} D_2(f)(x, y(x)) D_1(f)(x, y(x)) + \frac{1}{6} D_{1,1}(f)(x, y(x))\right) h^3 + \left(\frac{1}{2} D_2(f)(x, y(x)) f(x, y(x)) + \frac{1}{2} D_1(f)(x, y(x))\right) h^2 + f(x, y(x)) h + y(x) + E \quad (12)$$

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> NUMERIC_STEP := subs(y(x+h)=y[k+1],y(x)=y[k],subs(E=0,NEW_TAYLOR))
  ;
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$$NUMERIC\_STEP := y_{k+1} = \left(\frac{1}{24} D_{2,2,2}(f)(x, y_k) f(x, y_k)^3 + \frac{1}{6} f(x, y_k)^2 D_{2,2}(f)(x, y_k) D_2(f)(x, y_k) + \frac{1}{24} D_2(f)(x, y_k)^3 f(x, y_k) + \frac{1}{8} D_{1,2,2}(f)(x, y_k) f(x, y_k)^2 + \frac{1}{8} f(x, y_k) D_{2,2}(f)(x, y_k) D_1(f)(x, y_k) + \frac{5}{24} D_{1,2}(f)(x, y_k) D_2(f)(x, y_k) f(x, \quad (13)$$

$$y_k) + \frac{1}{24} D_2(f)(x, y_k)^2 D_1(f)(x, y_k) + \frac{1}{8} D_{1,1,2}(f)(x, y_k) f(x, y_k) + \frac{1}{8} D_{1,2}(f)(x, y_k) D_1(f)(x, y_k) + \frac{1}{24} D_2(f)(x, y_k) D_{1,1}(f)(x, y_k) + \frac{1}{24} D_{1,1,1}(f)(x, y_k)\Big) h^4 + \Big(\frac{1}{6} D_{2,2}(f)(x, y_k) f(x, y_k)^2 + \frac{1}{6} D_2(f)(x, y_k)^2 f(x, y_k) + \frac{1}{3} D_{1,2}(f)(x, y_k) f(x, y_k) + \frac{1}{6} D_2(f)(x, y_k) D_1(f)(x, y_k) + \frac{1}{6} D_{1,1}(f)(x, y_k)\Big) h^3 + \Big(\frac{1}{2} D_2(f)(x, y_k) f(x, y_k) + \frac{1}{2} D_1(f)(x, y_k)\Big) h^2 + f(x, y_k) h + y_k$$

Caso particolare f(x,y) = x*y+x^2

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> D1 := x*y+x^2 ;
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$$D1 := x^2 + x\,y \quad (14)$$

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> D2 := simplify(diff(D1,x)+diff(D1,y)*D1) ;
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$$D2 := x^3 + x^2\,y + 2\,x + y \quad (15)$$

```
> D3 := simplify(diff(D2,x)+diff(D2,y)*D1) ;
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$$D3 := x^4 + x^3\,y + 4\,x^2 + 3\,x\,y + 2 \quad (16)$$

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> D4 := simplify(diff(D3,x)+diff(D3,y)*D1) ;
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$$D4 := x^5 + x^4\,y + 7\,x^3 + 6\,x^2\,y + 8\,x + 3\,y \quad (17)$$

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> TAYLOR := y(x+h) = y(x) + D1 *h + D2 * h^2/2 + D3 *h^3/3! + D4 *
  h^4/4! + E ;
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$$TAYLOR := y(x+h) = y(x) + (x^2 + x\,y)\,h + \frac{1}{2}(x^3 + x^2\,y + 2\,x + y)\,h^2 + \frac{1}{6}(x^4 + x^3\,y + 4\,x^2 + 3\,x\,y + 2)\,h^3 + \frac{1}{24}(x^5 + x^4\,y + 7\,x^3 + 6\,x^2\,y + 8\,x + 3\,y)\,h^4 + E \quad (18)$$

```
> NUMERIC_STEP := subs(y(x+h)=y[k+1],y=y[k],x=x[k],subs(E=0,TAYLOR))
  ;
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$$NUMERIC\_STEP := y_{k_{k+1}} = y_k(x_k) + (x_k^2 + x_k\,y_k)\,h + \frac{1}{2}(x_k^3 + x_k^2\,y_k + 2\,x_k + y_k)\,h^2 + \frac{1}{6}(x_k^4 + x_k^3\,y_k + 4\,x_k^2 + 3\,x_k\,y_k + 2)\,h^3 + \frac{1}{24}(x_k^5 + x_k^4\,y_k + 7\,x_k^3 + 6\,x_k^2\,y_k + 8\,x_k + 3\,y_k)\,h^4 \quad (19)$$