Esempio metodo basato su Taylor

```
> restart:
> with(plots) :
```

y'=f(x,y) = x*y+x^2, y(0)=1

```
> D1 := -x*y+x^2 ;
```

$$D1 := x^2 - x\,y \tag{1}$$

```
> D2 := simplify(diff(D1,x)+diff(D1,y)*D1) ;
```

$$D2 := -x^3 + x^2\,y + 2\,x - y \tag{2}$$

```
> D3 := simplify(diff(D2,x)+diff(D2,y)*D1) ;
```

$$D3 := x^4 - x^3\,y - 4\,x^2 + 3\,x\,y + 2 \tag{3}$$

```
> D4 := simplify(diff(D3,x)+diff(D3,y)*D1) ;
```

$$D4 := -x^5 + x^4\,y + 7\,x^3 - 6\,x^2\,y - 8\,x + 3\,y \tag{4}$$

```
> D5 := simplify(diff(D4,x)+diff(D4,y)*D1) ;
```

$$D5 := x^6 - x^5\,y - 11\,x^4 + 10\,x^3\,y + 24\,x^2 - 15\,x\,y - 8 \tag{5}$$

```
> D6 := simplify(diff(D5,x)+diff(D5,y)*D1) ;
```

$$D6 := -x^7 + x^6\,y + 16\,x^5 - 15\,x^4\,y - 59\,x^3 + 45\,x^2\,y + 48\,x - 15\,y \tag{6}$$

```
> D7 := simplify(diff(D6,x)+diff(D6,y)*D1) ;
```

$$D7 := x^8 - x^7\,y - 22\,x^6 + 21\,x^5\,y + 125\,x^4 - 105\,x^3\,y - 192\,x^2 + 105\,x\,y + 48 \tag{7}$$

```
> D8 := simplify(diff(D7,x)+diff(D7,y)*D1) ;
```

$$D8 := -x^9 + x^8\,y + 29\,x^7 - 28\,x^6\,y - 237\,x^5 + 210\,x^4\,y + 605\,x^3 - 420\,x^2\,y - 384\,x + 105\,y \tag{8}$$

```
> TAYLOR1 := y(x+h) = y(x) + subs(y=y(x),D1) *h + E :
  TAYLOR2 := y(x+h) = rhs(TAYLOR1) + subs(y=y(x),D2)*h^2/2! :
  TAYLOR3 := y(x+h) = rhs(TAYLOR2) + subs(y=y(x),D3)*h^3/3! :
  TAYLOR4 := y(x+h) = rhs(TAYLOR3) + subs(y=y(x),D4)*h^4/4! :
  TAYLOR5 := y(x+h) = rhs(TAYLOR4) + subs(y=y(x),D5)*h^5/5! :
  TAYLOR6 := y(x+h) = rhs(TAYLOR5) + subs(y=y(x),D6)*h^6/6! :
  TAYLOR7 := y(x+h) = rhs(TAYLOR6) + subs(y=y(x),D7)*h^7/7! :
  TAYLOR8 := y(x+h) = rhs(TAYLOR7) + subs(y=y(x),D8)*h^8/8! ;
```

$$\begin{aligned} TAYLOR8 := y(x+h) = {} & y(x) + \left(x^2 - x\,y(x)\right) h + E + \frac{1}{2}\left(-x^3 + x^2\,y(x) + 2\,x - y(x)\right) h^2 \\ & + \frac{1}{6}\left(x^4 - x^3\,y(x) - 4\,x^2 + 3\,x\,y(x) + 2\right) h^3 + \frac{1}{24}\left(-x^5 + x^4\,y(x) + 7\,x^3 - 6\,x^2\,y(x) \right. \\ & \left. - 8\,x + 3\,y(x)\right) h^4 + \frac{1}{120}\left(x^6 - x^5\,y(x) - 11\,x^4 + 10\,x^3\,y(x) + 24\,x^2 - 15\,x\,y(x) \right. \\ & \left. - 8\right) h^5 + \frac{1}{720}\left(-x^7 + x^6\,y(x) + 16\,x^5 - 15\,x^4\,y(x) - 59\,x^3 + 45\,x^2\,y(x) + 48\,x \right. \\ & \left. - 15\,y(x)\right) h^6 + \frac{1}{5040}\left(x^8 - x^7\,y(x) - 22\,x^6 + 21\,x^5\,y(x) + 125\,x^4 - 105\,x^3\,y(x) \right. \\ & \left. - 192\,x^2 + 105\,x\,y(x) + 48\right) h^7 + \frac{1}{40320}\left(-x^9 + x^8\,y(x) + 29\,x^7 - 28\,x^6\,y(x) \right. \\ & \left. - 237\,x^5 + 210\,x^4\,y(x) + 605\,x^3 - 420\,x^2\,y(x) - 384\,x + 105\,y(x)\right) h^8 \end{aligned} \tag{9}$$

```
> SUBS := y(x+h)=y[k+1],y(x)=y[k],x=x[k],E=0 ;
```

$$SUBS := y(x+h) = y_{k+1}, y(x) = y_k, x = x_k, E = 0 \quad (10)$$

```
> NUMERIC_STEP1 := subs(SUBS,TAYLOR1) ;
  NUMERIC_STEP2 := subs(SUBS,TAYLOR2) :
  NUMERIC_STEP3 := subs(SUBS,TAYLOR3) :
  NUMERIC_STEP4 := subs(SUBS,TAYLOR4) :
  NUMERIC_STEP5 := subs(SUBS,TAYLOR5) :
  NUMERIC_STEP6 := subs(SUBS,TAYLOR6) :
  NUMERIC_STEP7 := subs(SUBS,TAYLOR7) :
  NUMERIC_STEP8 := subs(SUBS,TAYLOR8) ;
```

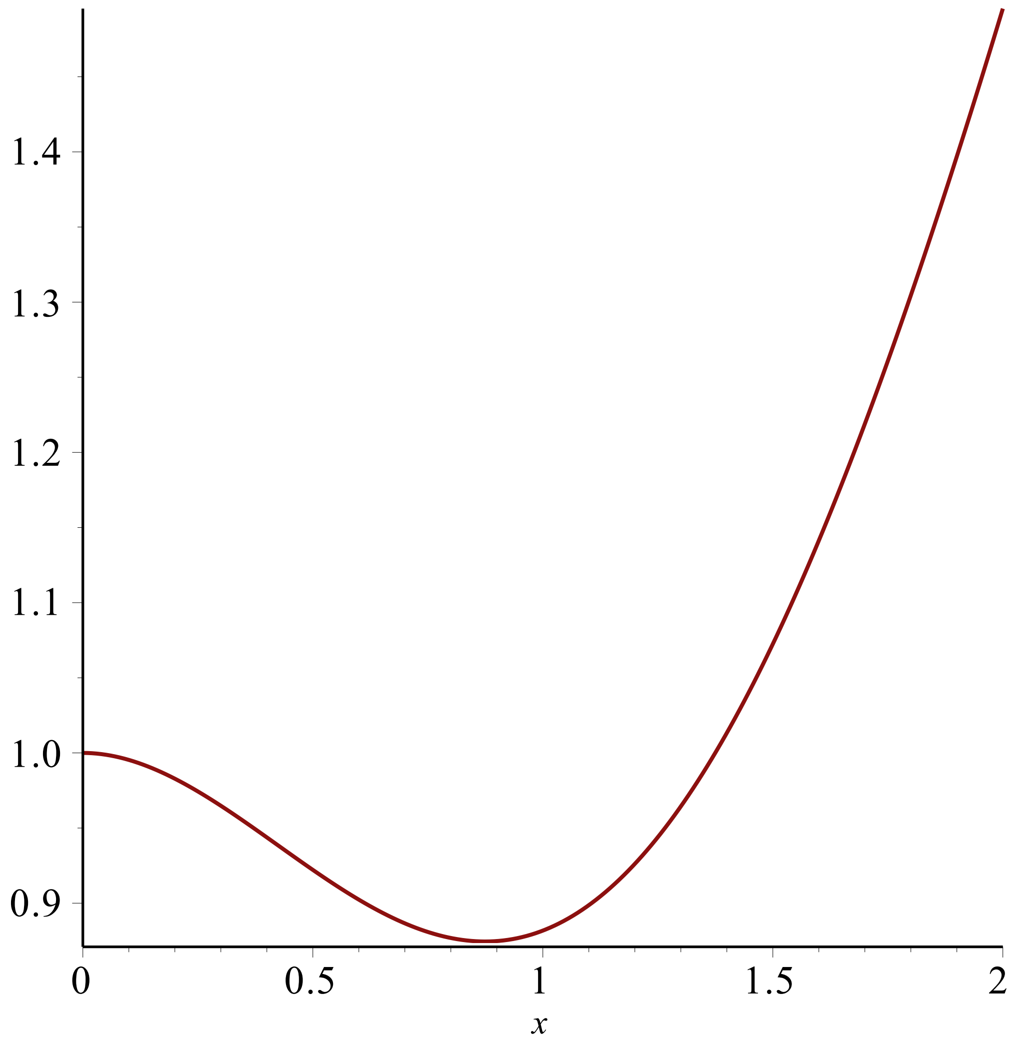
$$NUMERIC\_STEP1 := y_{k+1} = y_k + \left(x_k^2 - x_k y_k\right) h$$

$$NUMERIC\_STEP8 := y_{k+1} = y_k + \left(x_k^2 - x_k y_k\right) h + \frac{1}{2}\left(-x_k^3 + x_k^2 y_k + 2 x_k - y_k\right) h^2 + \frac{1}{6}\left(x_k^4 - x_k^3 y_k - 4 x_k^2 + 3 x_k y_k + 2\right) h^3 + \frac{1}{24}\left(-x_k^5 + x_k^4 y_k + 7 x_k^3 - 6 x_k^2 y_k - 8 x_k + 3 y_k\right) h^4 + \frac{1}{120}\left(x_k^6 - x_k^5 y_k - 11 x_k^4 + 10 x_k^3 y_k + 24 x_k^2 - 15 x_k y_k - 8\right) h^5 + \frac{1}{720}\left(-x_k^7 + x_k^6 y_k + 16 x_k^5 - 15 x_k^4 y_k - 59 x_k^3 + 45 x_k^2 y_k + 48 x_k - 15 y_k\right) h^6 + \frac{1}{5040}\left(x_k^8 - x_k^7 y_k - 22 x_k^6 + 21 x_k^5 y_k + 125 x_k^4 - 105 x_k^3 y_k - 192 x_k^2 + 105 x_k y_k + 48\right) h^7 + \frac{1}{40320}\left(-x_k^9 + x_k^8 y_k + 29 x_k^7 - 28 x_k^6 y_k - 237 x_k^5 + 210 x_k^4 y_k + 605 x_k^3 - 420 x_k^2 y_k - 384 x_k + 105 y_k\right) h^8 \quad (11)$$

```
> ESATTA := dsolve( {diff(y(x),x)=-x*y(x)+x^2,y(0)=1} ) ;
```

$$ESATTA := y(x) = x + \frac{1}{2}\,\mathrm{I}\,e^{-\frac{1}{2}x^2}\sqrt{\pi}\,\sqrt{2}\,\mathrm{erf}\left(\frac{1}{2}\,\mathrm{I}\sqrt{2}\,x\right) + e^{-\frac{1}{2}x^2} \quad (12)$$

```
> plot( subs(ESATTA,y(x)), x=0..2) ;
```

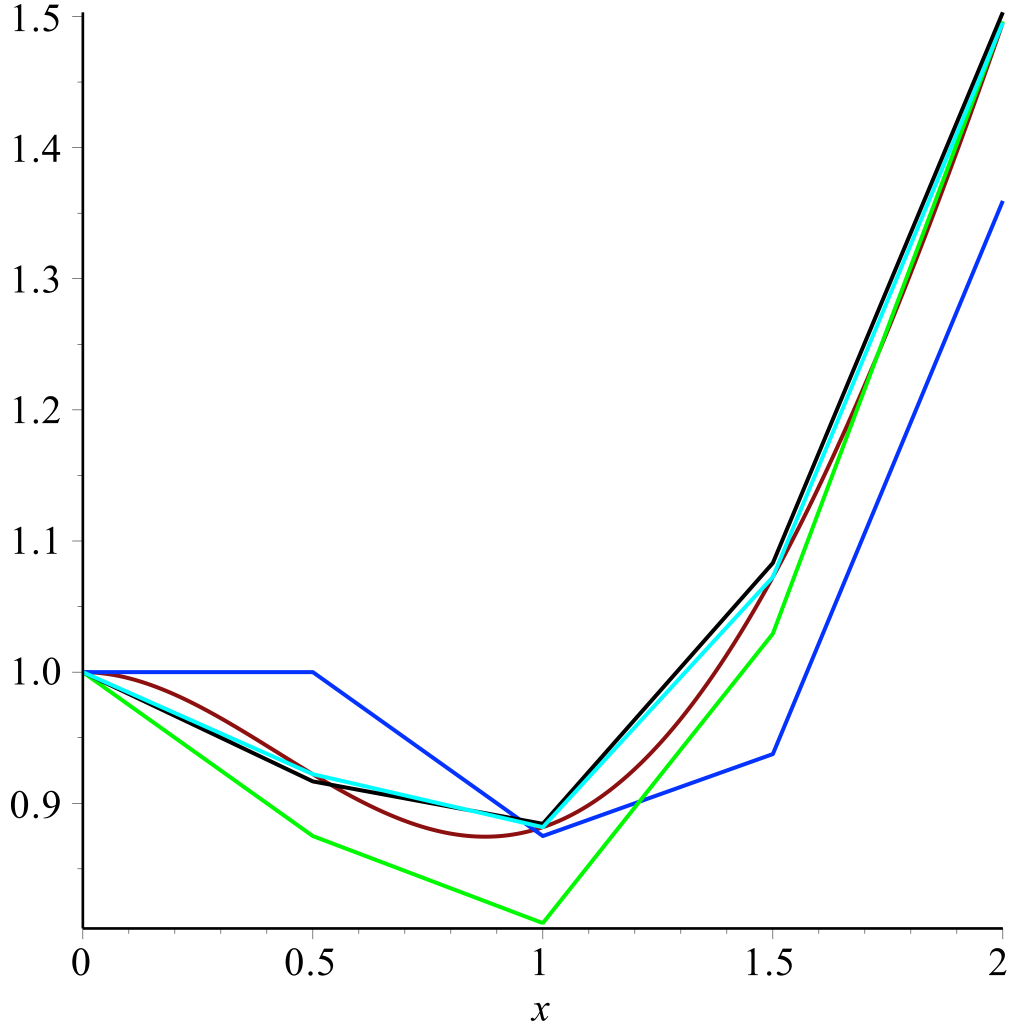



```
> h := 0.5 ;
```

$$h := 0.5 \tag{13}$$

```
> XY1 := [[0,1]]:
  XY2 := [[0,1]]:
  XY3 := [[0,1]]:
  XY8 := [[0,1]]:
  for j from 1 to 4 do
    XY1 := [op(XY1),[XY1[-1][1]+h,subs(x[k]=XY1[-1][1],y[k]=XY1[-1]
  [2],rhs(NUMERIC_STEP1))]]:

    XY2 := [op(XY2),[XY2[-1][1]+h,subs(x[k]=XY2[-1][1],y[k]=XY2[-1]
  [2],rhs(NUMERIC_STEP2))]]:
    XY3 := [op(XY3),[XY3[-1][1]+h,subs(x[k]=XY3[-1][1],y[k]=XY3[-1]
  [2],rhs(NUMERIC_STEP3))]]:
    XY8 := [op(XY8),[XY8[-1][1]+h,subs(x[k]=XY8[-1][1],y[k]=XY8[-1]
  [2],rhs(NUMERIC_STEP8))]]:
  end:
> E := plot(subs(ESATTA,y(x)), x=0..2):
> A1 := plot(XY1,color="blue"):
> A2 := plot(XY2,color="green"):
> A3 := plot(XY3,color="black"):
> A8 := plot(XY8,color="cyan"):
> display(E,A1,A2,A3,A8) ;
```




```
> YE := evalf(subs(x=2,subs(ESATTA,y(x)))) ;
```

$$YE := 1.495347209 \quad (14)$$

```
> XY1[-1][2]-YE ;
  XY2[-1][2]-YE ;
  XY3[-1][2]-YE ;
  XY8[-1][2]-YE ;
```

$$-0.135972209$$

$$0.000929647$$

$$0.007841949$$

$$-9.54\ 10^{-7} \quad (15)$$