$$\begin{cases} x_{k+1} = 2x_k + y_k - 1 & x_0 = 0 \\ y_{k+1} = 2y_k + w_k - k + 1 - 2^k & y_0 = 0 \\ w_{k+1} = w_k + y_k + 2^k & w_0 = 0 \end{cases}$$

Using Z - transform we have:

$$\begin{cases} (z-2)X - Y = -\frac{z}{z-1} & \Rightarrow \quad Y = \frac{z}{z-1} + (z-2)X \quad (I) \\ (z-2)Y - W = -\frac{z}{(z-1)^2} + \frac{z}{z-1} - \frac{z}{z-2} & \Rightarrow \quad W(z-1) = Y(z-1)(z-2) + \frac{z}{z-1} - z + \frac{z(z-1)}{z-2} \quad (II) \\ (z-1)W - z - Y - X = \frac{z}{z-2} & \end{cases}$$

substituting I and II in the last eqz.

$$(z-1)(z-2)\left(\frac{z}{z-1} + (z-2)X\right) + \cancel{\frac{z}{z-1}} - z + \frac{z(z-1)}{z-2} - z - \cancel{\frac{z}{z-1}} - (z-2)X - X = \frac{z}{z-2}$$

collecting X at L.H.S.

$$X \cdot \left[(z-1)(z-2)^2 - (z-2) - 1\right] = -z(z-2) + 2z - \frac{z(z-1)}{z-2} + \frac{z}{z-2}$$, simplifying yields

$$X \cdot \left[(z-1)(z-2)^2 - (z-1)\right] = \frac{-z}{z-2}\left((z-2)^2 - 2(z-2) + z - 1 - 1\right)$$; collected $-z$ and lcm

$$X \cdot \left[(z-1)(z^2 - 4z + 4 - 1)\right] = -\frac{z}{z-2}\left(z^2 - 4z + 4 - 2z + 4 + z - 2\right)$$

$$X \cdot \left[(z-1)(z-1)(z-3)\right] = -\frac{z}{z-2}(z^2 - 5z + 6) = -\frac{z}{z-2}(z-2)(z-3)$$, thus

$$X = -\frac{z(z-2)(z-3)}{(z-2)(z-1)^2(z-3)} = -\frac{z}{(z-1)^2} \quad \Rightarrow \quad Z^{-1}\left\{-\frac{z}{(z-1)^2}\right\} = -k$$

with the knowledge of X, we can compute Y from equation I, i.e.

$$Y = \frac{z}{z-1} + (z-2)X = \frac{z}{z-1} - (z-2)\frac{z}{(z-1)^2} = \frac{z(z-1) - z(z-2)}{(z-1)^2} = \frac{z(z-1-z+2)}{(z-1)^2} = \frac{z}{(z-1)^2}$$

hence $Z^{-1}\{Y\} = Z^{-1}\left\{\frac{z}{(z-1)^2}\right\} = k$. Substituting Y in equation II gives

$$W(z-1) = Y(z-1)(z-2) + \frac{z}{z-1} - z + \frac{z(z-1)}{z-2} \quad \Rightarrow \quad W = Y(z-2) + \frac{z}{(z-1)^2} - \frac{z}{z-1} + \frac{z}{z-2}$$

$$W = \frac{z(z-2)}{(z-1)^2} + \frac{z}{(z-1)^2} - \frac{z}{z-1} + \frac{z}{z-2} = \frac{z(z-2+1)}{(z-1)^2} - \frac{z}{z-1} + \frac{z}{z-2} =$$

$$= \frac{z}{z-1} - \frac{z}{z-1} + \frac{z}{z+2} = \frac{z}{z-2} \quad \Rightarrow \quad Z^{-1}\left\{\frac{z}{z-2}\right\} = 2^k$$. Therefore

$$\boxed{x_k = -k \quad , \quad y_k = k \quad , \quad w_k = 2^k}$$