

Partial Factor factorization

```
> R := (1+s+s^4+s^5)/(s*(s-1)*(s+2)*(s-3)*(s-4)*(s-10));
```

$$R := \frac{1 + s + s^4 + s^5}{s (s - 1) (s + 2) (s - 3) (s - 4) (s - 10)} \quad (1)$$

```
> convert(R,parfrac);
```

$$\frac{164}{105 (s - 3)} - \frac{1285}{432 (s - 4)} - \frac{2}{81 (s - 1)} + \frac{1}{240 s} + \frac{17}{2160 (s + 2)} + \frac{110011}{45360 (s - 10)} \quad (2)$$

Solution by "hand"

Coefficient for s

```
> alpha1 := subs(s=0,R*s) ;
```

$$\alpha_1 := \frac{1}{240} \quad (3)$$

Coefficient for s-1

```
> alpha2 := subs(s=1,R*(s-1)) ;
```

$$\alpha_2 := -\frac{2}{81} \quad (4)$$

Coefficient for s+2

```
> alpha3 := subs(s=-2,R*(s+2)) ;
```

$$\alpha_3 := \frac{17}{2160} \quad (5)$$

Coefficient for s-3

```
> alpha4 := subs(s=3,R*(s-3)) ;
```

$$\alpha_4 := \frac{164}{105} \quad (6)$$

Coefficient for s-4

```
> alpha5 := subs(s=4,R*(s-4)) ;
```

$$\alpha_5 := -\frac{1285}{432} \quad (7)$$

Coefficient for s-10

```
> alpha6 := subs(s=10,R*(s-10)) ;
```

$$\alpha_6 := \frac{110011}{45360} \quad (8)$$

```
> Rcheck := alpha1/s + alpha2/(s-1) + alpha3/(s+2) + alpha4/(s-3) +  
alpha5/(s-4) + alpha6/(s-10);
```

$$Rcheck := \frac{164}{105 (s - 3)} - \frac{1285}{432 (s - 4)} - \frac{2}{81 (s - 1)} + \frac{1}{240 s} + \frac{17}{2160 (s + 2)} + \frac{110011}{45360 (s - 10)} \quad (9)$$

```
> simplify(R-Rcheck) ;
```

$$0 \quad (10)$$

Partial fraction expansion

```
> restart;
```

```
> R := (1+s+s^3)/(s-1)^2/s/(s+3) ;
```

$$R := \frac{1 + s + s^3}{(s - 1)^2 s (s + 3)} \quad (11)$$

The expansion must be of the form

```
> Rexp := alpha1/s + alpha2/(s+3) + alpha3/(s-1) + alpha4/(s-1)^2 ;
```

$$Rexp := \frac{\alpha_1}{s} + \frac{\alpha_2}{s + 3} + \frac{\alpha_3}{s - 1} + \frac{\alpha_4}{(s - 1)^2} \quad (12)$$

Trivial (stupid) method: expand and match

```
> numer(R-Rexp)/denom(R-Rexp) ;
```

$$\frac{1}{(s - 1)^2 s (s + 3)} (1 + s + s^3 - \alpha_1 s^3 - \alpha_1 s^2 + 5 \alpha_1 s - 3 \alpha_1 - \alpha_2 s^3 + 2 \alpha_2 s^2 - \alpha_2 s - \alpha_3 s^3 - 2 \alpha_3 s^2 + 3 \alpha_3 s - \alpha_4 s^2 - 3 \alpha_4 s) \quad (13)$$

```
> POLY := collect(numer(R-Rexp), s) ;
```

$$POLY := (-\alpha_1 + 1 - \alpha_2 - \alpha_3) s^3 + (-2 \alpha_3 - \alpha_1 + 2 \alpha_2 - \alpha_4) s^2 + (1 + 3 \alpha_3 + 5 \alpha_1 - 3 \alpha_4 - \alpha_2) s + 1 - 3 \alpha_1 \quad (14)$$

Select the equation for the linear system

```
> EQ1 := subs(s=0, POLY) ;
```

```
EQ2 := subs(s=0, diff(POLY, s)) ;
```

```
EQ3 := subs(s=0, diff(POLY, s, s)/2) ;
```

```
EQ4 := subs(s=0, diff(POLY, s, s, s)/6) ;
```

$$EQ1 := 1 - 3 \alpha_1$$

$$EQ2 := 1 + 3 \alpha_3 + 5 \alpha_1 - 3 \alpha_4 - \alpha_2$$

$$EQ3 := -2 \alpha_3 - \alpha_1 + 2 \alpha_2 - \alpha_4$$

$$EQ4 := -\alpha_1 + 1 - \alpha_2 - \alpha_3 \quad (15)$$

```
> solve( {EQ1 || (1..4)}, {alpha || (1..4)} ) ;
```

$$\left\{ \alpha_1 = \frac{1}{3}, \alpha_2 = \frac{29}{48}, \alpha_3 = \frac{1}{16}, \alpha_4 = \frac{3}{4} \right\} \quad (16)$$

Evaluate coeffs using "intelligent thinking"

```
> subs(s=0, R*s) ;
```

```
subs(s=-3, R*(s+3)) ;
```

```
subs(s=1, R*(s-1)^2) ;
```

$$\frac{1}{3}$$

$$\frac{29}{48}$$

$$\frac{3}{4} \quad (17)$$

```
> diff(R*(s-1)^2,s); simplify(%);
```

$$\frac{1+3s^2}{s(s+3)} - \frac{1+s+s^3}{s^2(s+3)} - \frac{1+s+s^3}{s(s+3)^2} - \frac{-s^2-2s+s^4+6s^3-3}{s^2(s+3)^2}$$

(18)

```
> subs(s=1,diff(R*(s-1)^2,s));
```

$$\frac{1}{16}$$

(19)

Using "deflaction"

Find coeffs for s

```
> alpha1 := subs(s=0,R*s);
```

$$\alpha_1 := \frac{1}{3}$$

(20)

```
> R-alpha1/s; simplify(%); R1 := %;
```

$$\frac{1+s+s^3}{(s-1)^2 s (s+3)} - \frac{1}{3s} - \frac{1}{3} \frac{8+2s^2-s}{(s-1)^2 (s+3)} - \frac{1}{3} \frac{8+2s^2-s}{(s-1)^2 (s+3)}$$

(21)

```
> alpha2 := subs(s=-3,R1*(s+3));
```

$$\alpha_2 := \frac{29}{48}$$

(22)

```
> R1-alpha2/(s+3); simplify(%); R2 := %;
```

$$\frac{1}{3} \frac{8+2s^2-s}{(s-1)^2 (s+3)} - \frac{29}{48(s+3)} - \frac{1}{16} \frac{s+11}{(s-1)^2} - \frac{1}{16} \frac{s+11}{(s-1)^2}$$

(23)

```
> alpha4 := subs(s=1,R2*(s-1)^2);
```

$$\alpha_4 := \frac{3}{4}$$

(24)

```
> R2-alpha4/(s-1)^2; simplify(%); R3 := %;
```

$$\frac{1}{16} \frac{s+11}{(s-1)^2} - \frac{3}{4(s-1)^2} - \frac{1}{16(s-1)}$$

(25)

$$R3 := \frac{1}{16(s-1)} \quad (25)$$

```
> alpha3 := subs(s=1,R3*(s-1)) ;
```

$$\alpha3 := \frac{1}{16} \quad (26)$$

```
> R3-alpha3/(s-1) ; simplify(%) ; R4 := % ;
```

0

0

$$R4 := 0 \quad (27)$$