Fourier Serie Expansion Example

Triangular wave

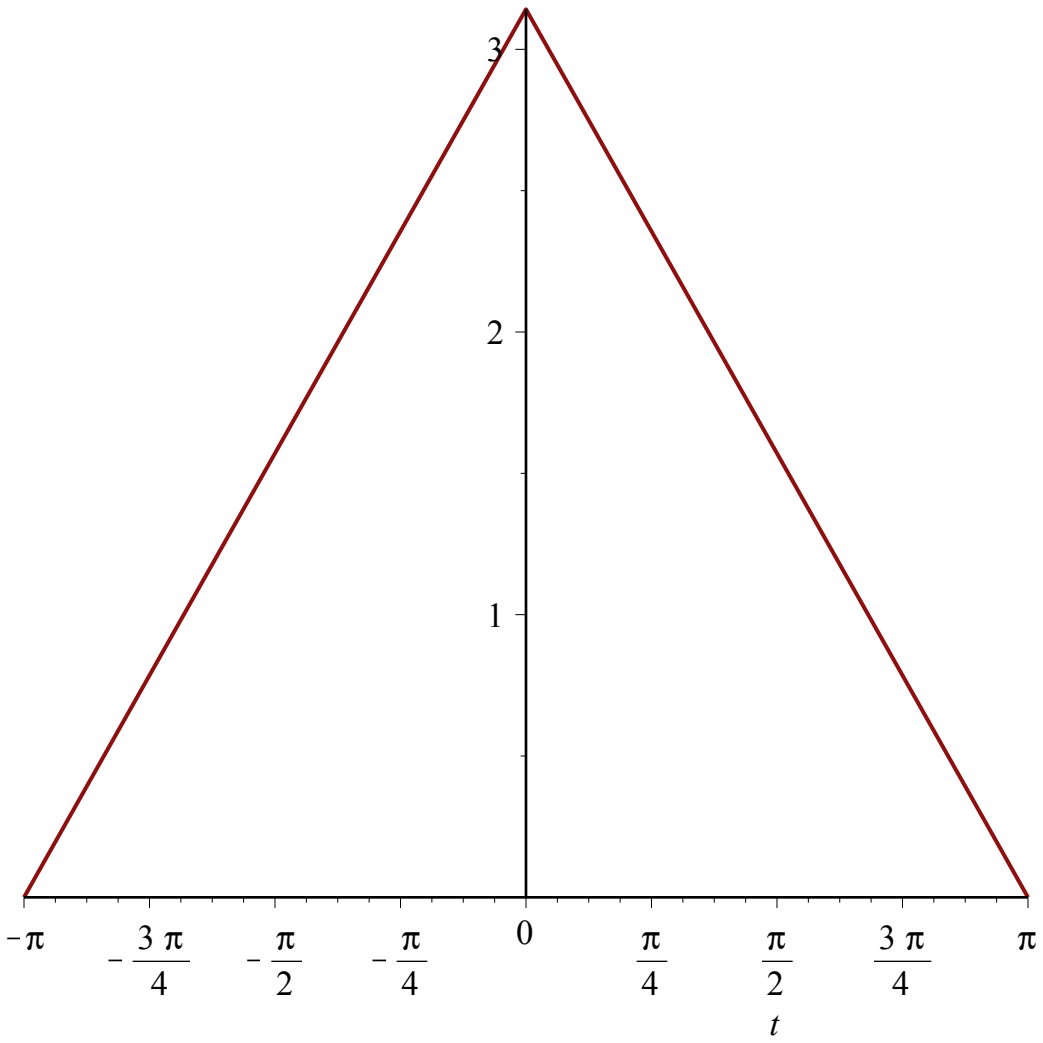
```
> restart:
> S := unapply( a0/2 + sum( ak(k)*cos(k*t) + bk(k)*sin(k*t), k=1..
  N), N) ;
```

$$S := N \to -\frac{1}{\pi N}\left(\mathrm{I}\left(\ln\left(\mathrm{e}^{\mathrm{I}t}+1\right)N+(-1)^{N}\mathrm{e}^{\mathrm{I}tN}\,\mathrm{LerchPhi}\left(-\mathrm{e}^{\mathrm{I}t},1,N\right)N-(-1)^{N}\mathrm{e}^{\mathrm{I}tN}\right.\right.$$
$$\left.\left.-\mathrm{e}^{-\mathrm{I}tN}(-1)^{N}\,\mathrm{LerchPhi}\left(-\mathrm{e}^{-\mathrm{I}t},1,N\right)N-\ln\left(1+\mathrm{e}^{-\mathrm{I}t}\right)N+\mathrm{e}^{-\mathrm{I}tN}(-1)^{N}\right)\right)$$
$$+\frac{1}{\pi N}\left(\mathrm{I}\left(\ln\left(1-\mathrm{e}^{\mathrm{I}t}\right)N+\mathrm{e}^{\mathrm{I}tN}\,\mathrm{LerchPhi}\left(\mathrm{e}^{\mathrm{I}t},1,N\right)N-\mathrm{e}^{\mathrm{I}tN}\right.\right.$$
$$\left.\left.-\mathrm{e}^{-\mathrm{I}tN}\,\mathrm{LerchPhi}\left(\mathrm{e}^{-\mathrm{I}t},1,N\right)N-\ln\left(1-\mathrm{e}^{-\mathrm{I}t}\right)N+\mathrm{e}^{-\mathrm{I}tN}\right)\right) \quad (1.1)$$

```
> f := piecewise( t<0, Pi+t, Pi-t ) ;
```

$$f := \begin{cases} \pi + t & t < 0 \\ \pi - t & otherwise \end{cases} \quad (1.2)$$

```
> plot( f, t=-Pi..Pi ) ;
```




```
> a0 := int( f, t=-Pi..Pi ) / Pi ;
```

$$a0 := \pi \tag{1.3}$$

```
> int( f*cos(k*t), t=-Pi..Pi ) / Pi ;
  ak := unapply( simplify( % ), k ) assuming k::integer ;
```

$$-\frac{2\left(\cos\left(\pi k\right)-1\right)}{k^2\,\pi}$$

$$ak := k \rightarrow \frac{2\left(\left(-1\right)^{k+1}+1\right)}{k^2\,\pi} \tag{1.4}$$

```
> int( f*sin(k*t), t=-Pi..Pi ) / Pi ;
  bk := unapply( simplify( % ), k ) assuming k::integer ;
```

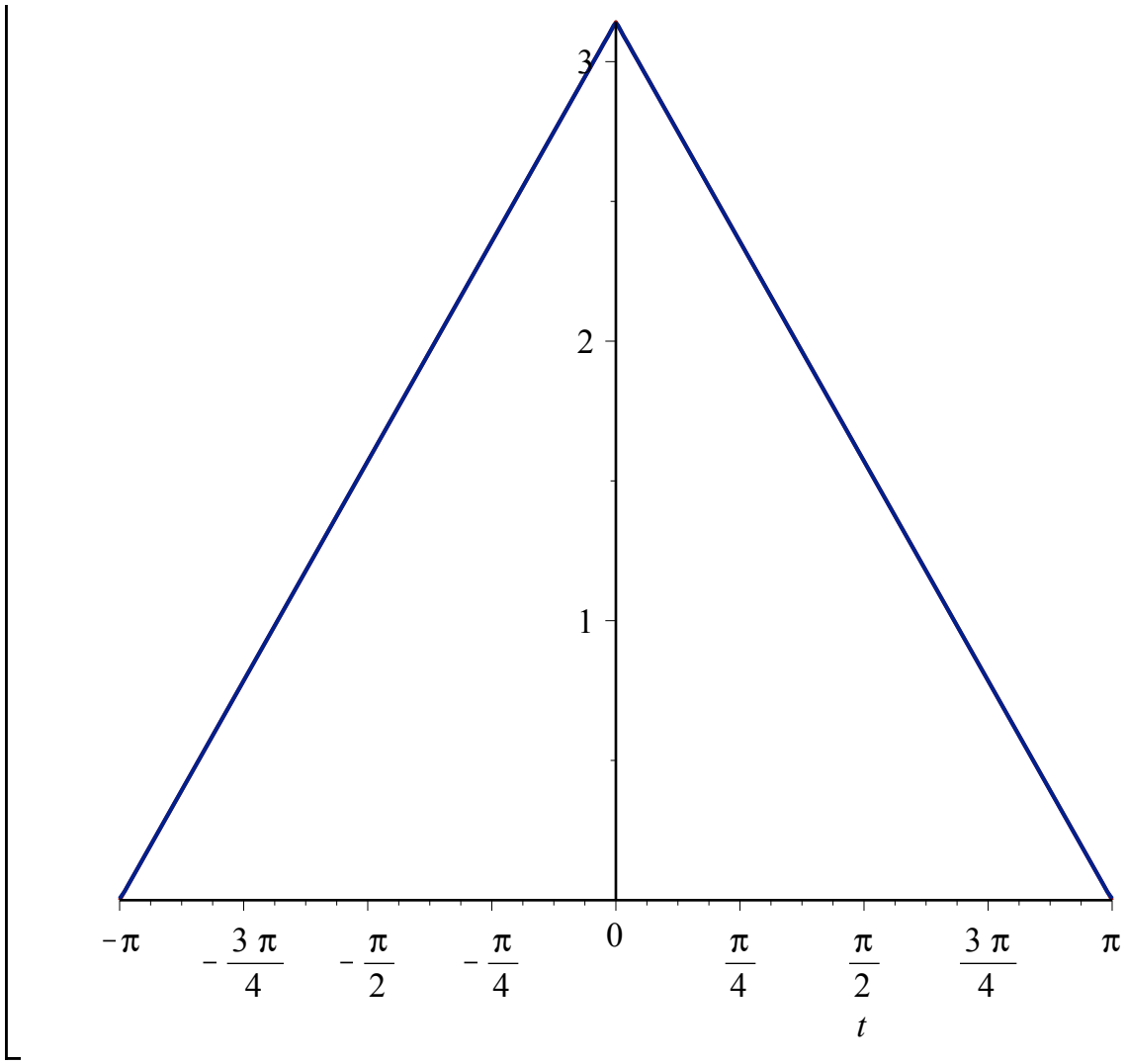
$$0$$

$$bk := k \rightarrow 0 \tag{1.5}$$

```
> S(5) ;
```

$$\frac{1}{2}\,\pi + \frac{4\cos\left(t\right)}{\pi} + \frac{4}{9}\,\frac{\cos\left(3\,t\right)}{\pi} + \frac{4}{25}\,\frac{\cos\left(5\,t\right)}{\pi} \tag{1.6}$$

```
> plot( [f,S(101)], t=-Pi..Pi ) ;
```



Square wave

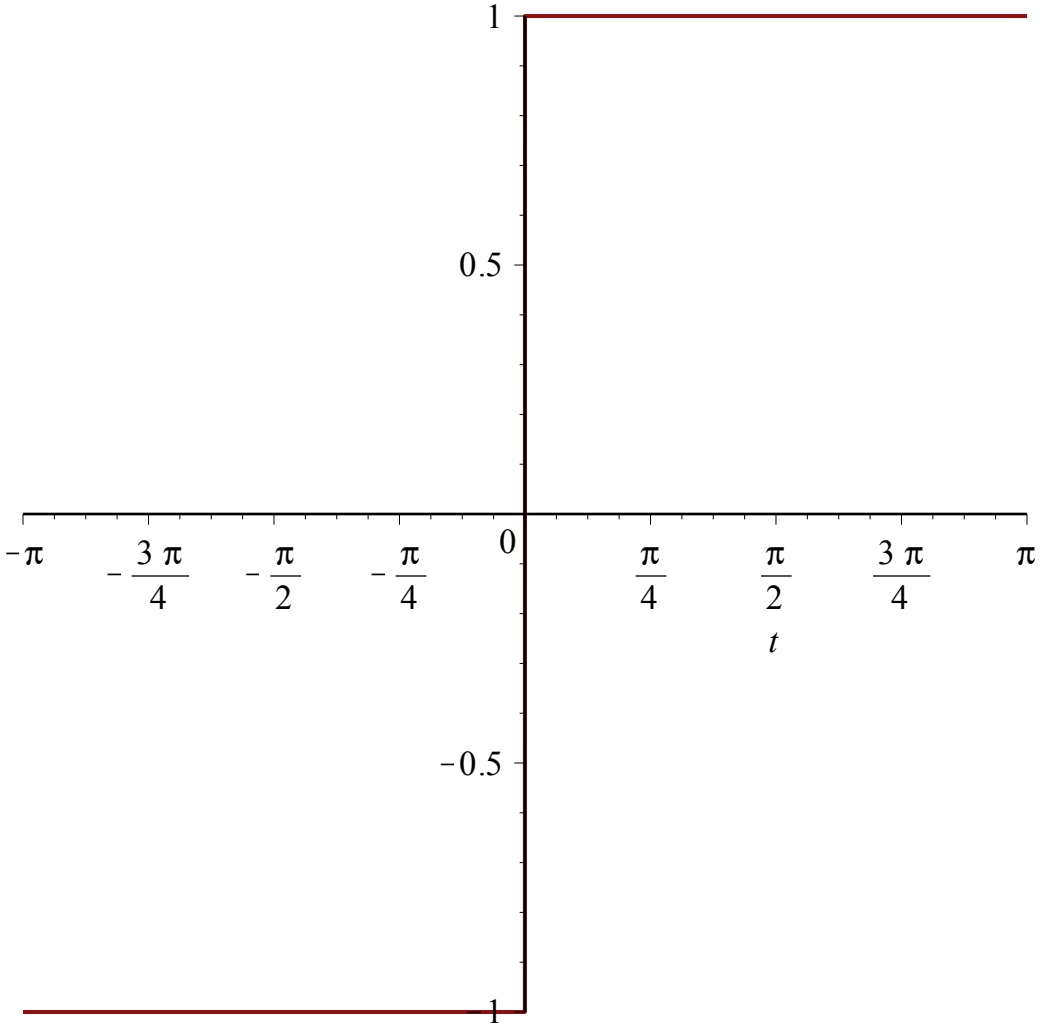
```
> restart:
> S := unapply( a0/2 + sum( ak(k)*cos(k*t) + bk(k)*sin(k*t), k=1..
  N), N) ;
```

$$S := N \rightarrow \frac{1}{2}\, a0 + \sum_{k=1}^{N} \left(ak(k) \cos(k\,t) + bk(k) \sin(k\,t) \right) \tag{2.1}$$

```
> f := piecewise( t<0, -1, 1 ) ;
```

$$f := \begin{cases} -1 & t < 0 \\ 1 & otherwise \end{cases} \tag{2.2}$$

```
> plot( f, t=-Pi..Pi ) ;
```




```
> a0 := int( f, t=-Pi..Pi ) / Pi ;
```

$$a0 := 0 \tag{2.3}$$

```
> int( f*cos(k*t), t=-Pi..Pi ) / Pi ;
  ak := unapply( simplify( % ), k ) assuming k::integer ;
```

$$0$$

$$ak := k \rightarrow 0 \tag{2.4}$$

```
> int( f*sin(k*t), t=-Pi..Pi ) / Pi ;
  bk := unapply( simplify( % ), k ) assuming k::integer ;
```

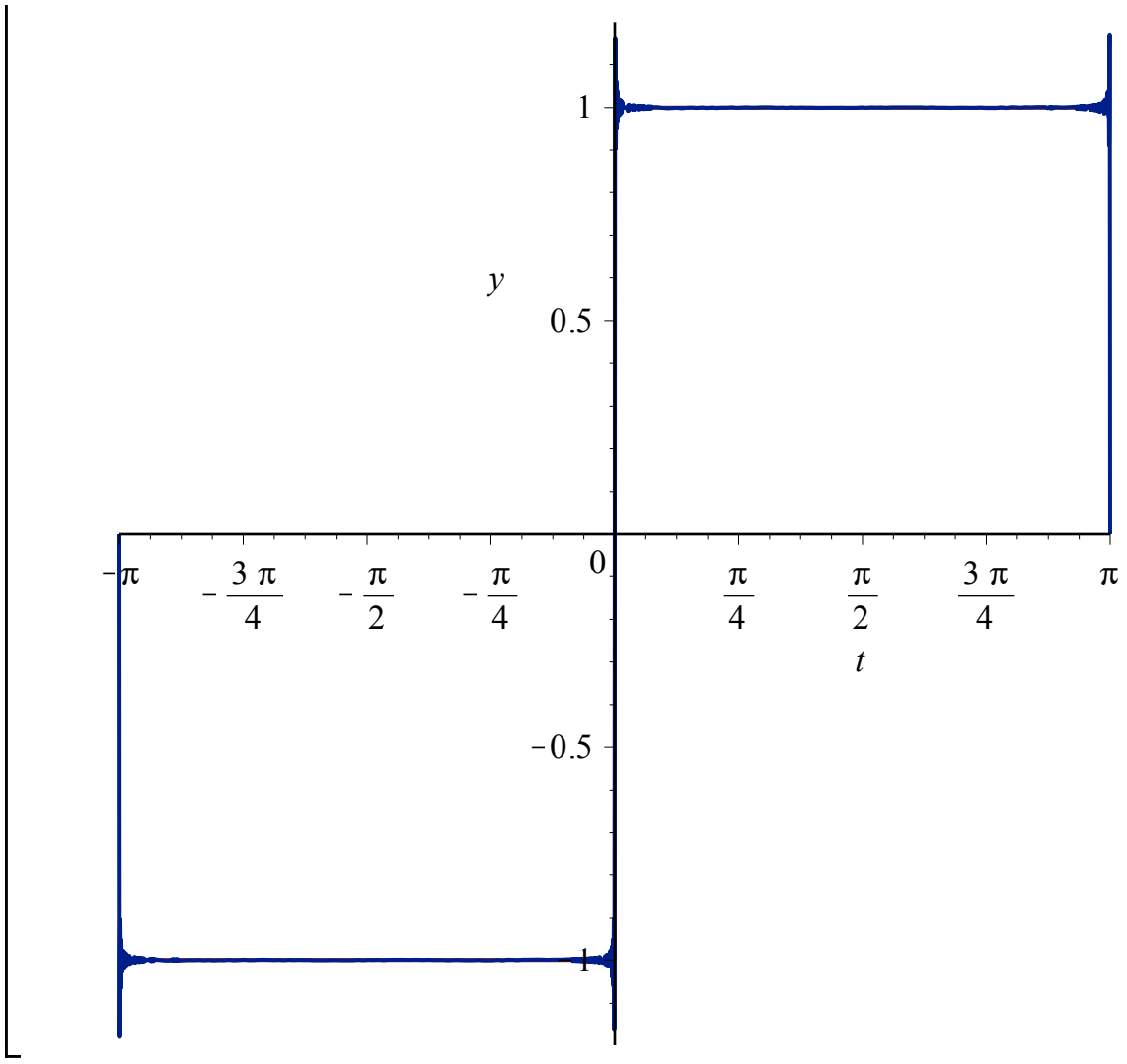
$$-\frac{2\left(\cos\left(\pi k\right)-1\right)}{k\pi}$$

$$bk := k \rightarrow \frac{2\left(\left(-1\right)^{k+1}+1\right)}{k\pi} \tag{2.5}$$

```
> S(5) ;
```

$$\frac{4\sin(t)}{\pi}+\frac{4}{3}\,\frac{\sin(3t)}{\pi}+\frac{4}{5}\,\frac{\sin(5t)}{\pi} \tag{2.6}$$

```
> plot( [f,S(1001)], t=-Pi..Pi, y=-1.2..1.2 ) ;
```



Half sin

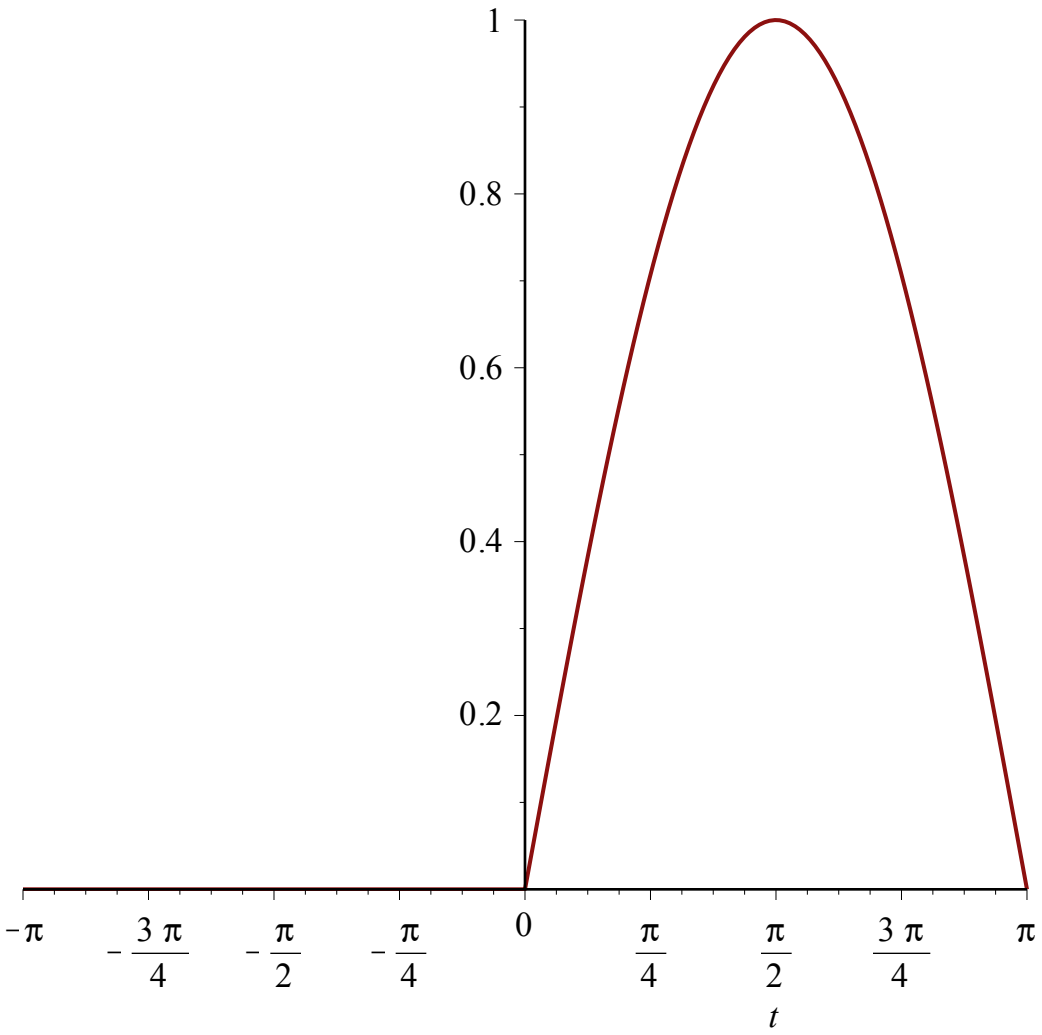
```
> restart:
> S := unapply( a0/2 + sum( ak(k)*cos(k*t) + bk(k)*sin(k*t), k=1..
  N), N) ;
```

$$S := N \rightarrow \frac{1}{2}\, a0 + \sum_{k=1}^{N} \left(ak(k) \cos(k\,t) + bk(k) \sin(k\,t) \right) \tag{3.1}$$

```
> f := piecewise( t<0, 0, sin(t) ) ;
```

$$f := \begin{cases} 0 & t < 0 \\ \sin(t) & otherwise \end{cases} \tag{3.2}$$

```
> plot( f, t=-Pi..Pi ) ;
```




```
> a0 := int( f, t=-Pi..Pi ) / Pi ;
```

$$a0 := \frac{2}{\pi} \tag{3.3}$$

```
> int( f*cos(t), t=-Pi..Pi ) / Pi ;
```

$$0 \tag{3.4}$$

```
> int( f*cos(k*t), t=-Pi..Pi ) / Pi ;
  ak := unapply( simplify( % ), k ) assuming k::integer ;
```

$$-\frac{1+\cos(\pi k)}{(k^2-1)\pi}$$

$$ak := k \rightarrow -\frac{1+(-1)^k}{(k^2-1)\pi} \tag{3.5}$$

```
> ak := unapply( piecewise( k<2, 0, -(1+(-1)^k)/((k^2-1)*Pi) ), k);
```

$$ak := k \rightarrow piecewise\left(k < 2, 0, -\frac{1+(-1)^k}{(k^2-1)\pi}\right) \tag{3.6}$$

```
> int( f*sin(t), t=-Pi..Pi ) / Pi ;
```

(3.7)

$$\frac{1}{2} \tag{3.7}$$

```
> int( f*sin(k*t), t=-Pi..Pi ) / Pi ;
  bk := unapply( simplify( % ), k ) assuming k::integer ;
```

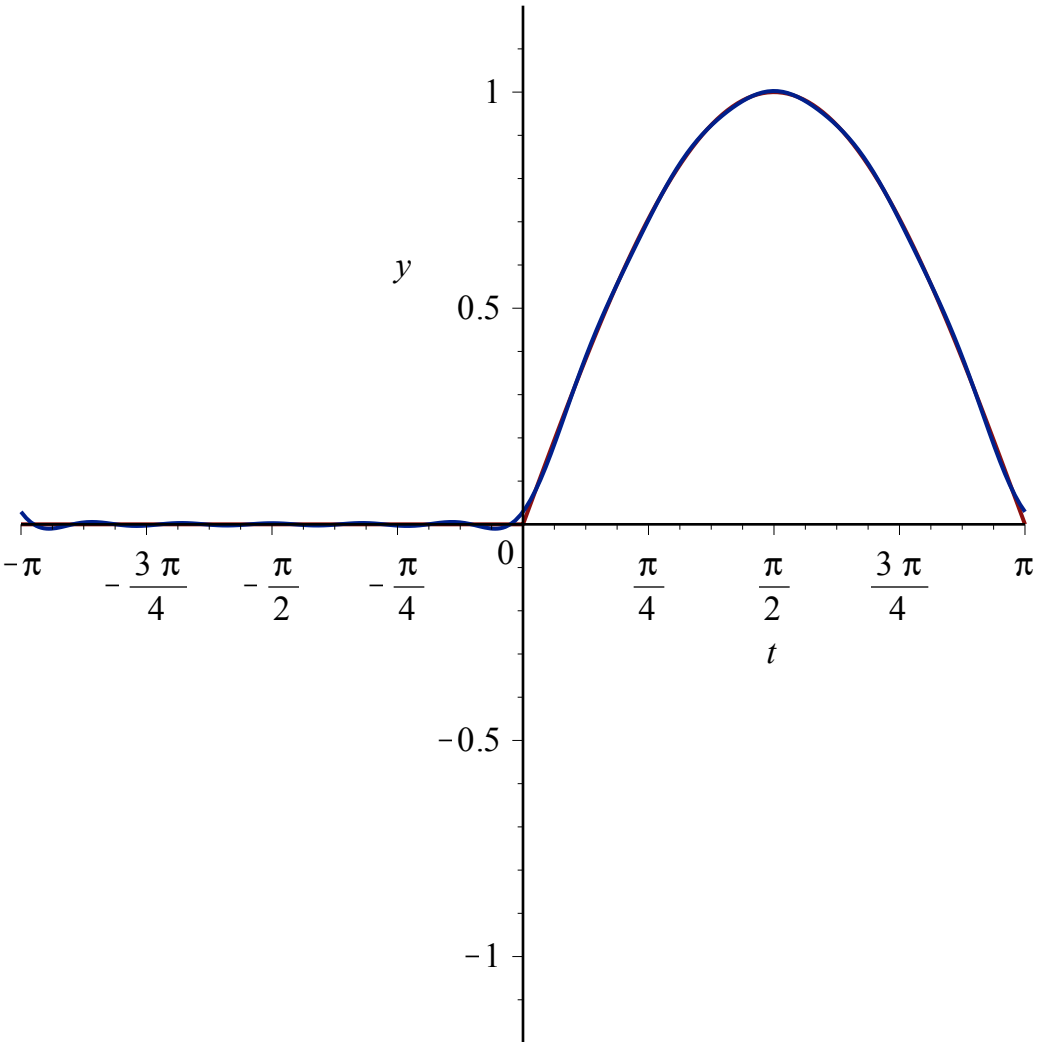
$$-\frac{\sin(\pi k)}{(k^2-1)\pi}$$

$$bk := k \to 0 \tag{3.8}$$

```
> bk := unapply( piecewise( k<2, 1/2,0), k);
```

$$bk := k \to piecewise\left(k < 2, \frac{1}{2}, 0\right) \tag{3.9}$$

```
> plot( [f,S(10)], t=-Pi..Pi, y=-1.2..1.2 ) ;
```




```
> diff(log(1+x)/6!,x,x,x,x,x,x) ;
```

$$-\frac{1}{6\,(1+x)^6} \tag{3.10}$$