

Solving Chong Zak problem

```
> restart:
with(LinearAlgebra):
```

Function to minimize

```
> f := -R1*i^2 ;
```

$$f := -RI i^2$$

(1)

Two constraints

```
> g := R1 ;
h := 20 - (R1+10)*i ;
```

$$g := RI$$

$$h := 20 - (RI + 10) i$$

(2)

Build the Lagrangian

```
> L := f - lambda*h - mu*g ;
```

$$L := -RI i^2 - \lambda (20 - (RI + 10) i) - \mu RI$$

(3)

Build the nonlinear system with KKT conditions

```
> EQ1 := diff(L,R1) ;
EQ2 := diff(L,i) ;
EQ3 := h ;
EQ4 := g * mu ;
```

$$EQ1 := -i^2 + \lambda i - \mu$$

$$EQ2 := -2 RI i - \lambda (-RI - 10)$$

$$EQ3 := 20 - (RI + 10) i$$

$$EQ4 := \mu RI$$

(4)

Solve the nonlinear system and discard solution with mu < 0.

```
> SOLi := solve( EQ3, {i} ) ;
```

$$SOLi := \left\{ i = \frac{20}{RI + 10} \right\}$$

(5)

```
> SOLl := solve( EQ2, {lambda} ) ;
```

$$SOLl := \left\{ \lambda = \frac{2 RI i}{RI + 10} \right\}$$

(6)

```
> SOLR1 := subs( SOLi, SOLl ) ;
```

$$SOLR1 := \left\{ \lambda = \frac{40 RI}{(RI + 10)^2} \right\}$$

(7)

Case when R1 = 0

```
> solve( subs(R1=0, {EQ1 || (1..4)} ), {i, lambda, mu} ) ;
```

$$\{i = 2, \lambda = 0, \mu = -4\}$$

(8)

mu is negative --> solution to be discarded

In this case lambda = 0, i = 2 and mu > 0. Ok

Case mu = 0

```
> subs(SOLi, subs(SOL1, subs(mu=0, SOLi, EQ1))) ; EQR1 := simplify(%);
```

$$-\frac{400}{(RI+10)^2} + \frac{800RI}{(RI+10)^3}$$

$$EQRI := \frac{400(RI-10)}{(RI+10)^3} \quad (9)$$

```
> SOLR1 := solve(EQR1, {R1}) ;
```

$$SOLR1 := \{RI = 10\} \quad (10)$$

```
> solve(subs(SOLR1, mu=0, {EQ1(1..4)}), {i, lambda}) ;
```

$$\{i = 1, \lambda = 1\} \quad (11)$$

We found 1 solution:

```
> SOL := R1=10, i=1, lambda=1, mu=0 ;
```

$$SOL := RI = 10, i = 1, \lambda = 1, \mu = 0 \quad (12)$$

Compute the hessian

```
> Hess := <<diff(L, R1, R1), diff(L, R1, i)> | <diff(L, R1, i), diff(L, i, i)>>;
```

$$Hess := \begin{bmatrix} 0 & -2i + \lambda \\ -2i + \lambda & -2RI \end{bmatrix} \quad (13)$$

```
> Hess1 := subs(SOL, Hess) ;
```

$$Hess1 := \begin{bmatrix} 0 & -1 \\ -1 & -20 \end{bmatrix} \quad (14)$$

g(R1) is not active

```
> H := <diff(h, R1) | diff(h, i)> ;
```

$$H := \begin{bmatrix} -i & -RI - 10 \end{bmatrix} \quad (15)$$

```
> H1 := subs(SOL, H) ;
```

$$H1 := \begin{bmatrix} -1 & -20 \end{bmatrix} \quad (16)$$

```
> Z1 := op(NullSpace(H1)) ;
```

$$Z1 := \begin{bmatrix} -20 \\ 1 \end{bmatrix} \quad (17)$$

```
> Transpose(Z1) . Hess1 . Z1 ;
```

$$20 \quad (18)$$