

Solve a DAE using non stabilized numerical schemes

> restart;

The DAE

> EQ1 := x(t)-sin(t) ;

EQ2 := diff(x(t),t)+y(t) ;

$$EQ1 := x(t) - \sin(t)$$

$$EQ2 := \frac{d}{dt} x(t) + y(t) \quad (1)$$

Reduce the index

> DEQ1 := diff(EQ1,t) ;

$$DEQ1 := \frac{d}{dt} x(t) - \cos(t) \quad (2)$$

> subs(solve(DEQ1,{diff(x(t),t)},EQ2),EQ2) ;

DEQ2 := diff(%,t) ;

$$\cos(t) + y(t)$$

$$DEQ2 := -\sin(t) + \frac{d}{dt} y(t) \quad (3)$$

Approximate solution of the ODE by using Explicit Euler

> ODE := {DEQ1,DEQ2} ;

$$ODE := \left\{ \frac{d}{dt} x(t) - \cos(t), -\sin(t) + \frac{d}{dt} y(t) \right\} \quad (1.1)$$

> SUBS := diff(x(t), t) = (x[k+1]-x[k])/h,

diff(y(t), t) = (y[k+1]-y[k])/h,

cos(t) = cos(k*h),

sin(t) = sin(k*h);

$$SUBS := \frac{d}{dt} x(t) = \frac{x_{k+1} - x_k}{h}, \frac{d}{dt} y(t) = \frac{y_{k+1} - y_k}{h}, \cos(t) = \cos(kh), \sin(t) \quad (1.2)$$

$$= \sin(kh)$$

> ExplicitEuler := subs(SUBS,ODE) ;

$$ExplicitEuler := \left\{ \frac{x_{k+1} - x_k}{h} - \cos(kh), -\sin(kh) + \frac{y_{k+1} - y_k}{h} \right\} \quad (1.3)$$

> ExplicitEulerAdvance := solve(ExplicitEuler, {x[k+1],y[k+1]}) ;

$$ExplicitEulerAdvance := \{x_{k+1} = x_k + \cos(kh)h, y_{k+1} = \sin(kh)h + y_k\} \quad (1.4)$$

> T := 10.5*Pi ;

N := 500 ;

h := T/N ;

$$T := 10.5 \pi$$

$$N := 500$$

$$h := 0.02100000000 \pi \quad (1.5)$$

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> X := [0.1] ;  
Y := [1] ;  
for k from 1 to N do  
  SOLSTEP := subs(x[k]=X[-1],y[k]=Y[-1],ExplicitEulerAdvance):  
  X := [op(X),evalf(subs( SOLSTEP, x[k+1] ))] ;  
  Y := [op(Y),evalf(subs( SOLSTEP, y[k+1] ))] ;  
end:
```

```
X:= [0.1]
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```
Y:= [1]
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(1.6)

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> plot( [seq([X[k],Y[k]],k=1..N)] );
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