Solving INDEX-1 DAE

```
> restart:
> EQ1 := diff(x(t),t) - x(t)*y(t) ;
```

$$EQ1 := \frac{d}{dt}\, x(t) - x(t)\, y(t) \tag{1}$$

```
> ALG1 := x(t)^2 + y(t) + 1 ;
```

$$ALG1 := x(t)^2 + y(t) + 1 \tag{2}$$

Solve the constraint

```
> SOL := solve( ALG1, {y(t)} ) ;
```

$$SOL := \{y(t) = -x(t)^2 - 1\} \tag{3}$$

```
> subs( SOL, EQ1 ) ;
```

$$\frac{d}{dt}\, x(t) - x(t)\, (-x(t)^2 - 1) \tag{4}$$

```
> dsolve( % ) ;
```

$$x(t) = \frac{1}{\sqrt{-1 + e^{2t}\, \_C1}},\ x(t) = -\frac{1}{\sqrt{-1 + e^{2t}\, \_C1}} \tag{5}$$

In practice in general y(t) cannot be solved

The implicit Euler Step is

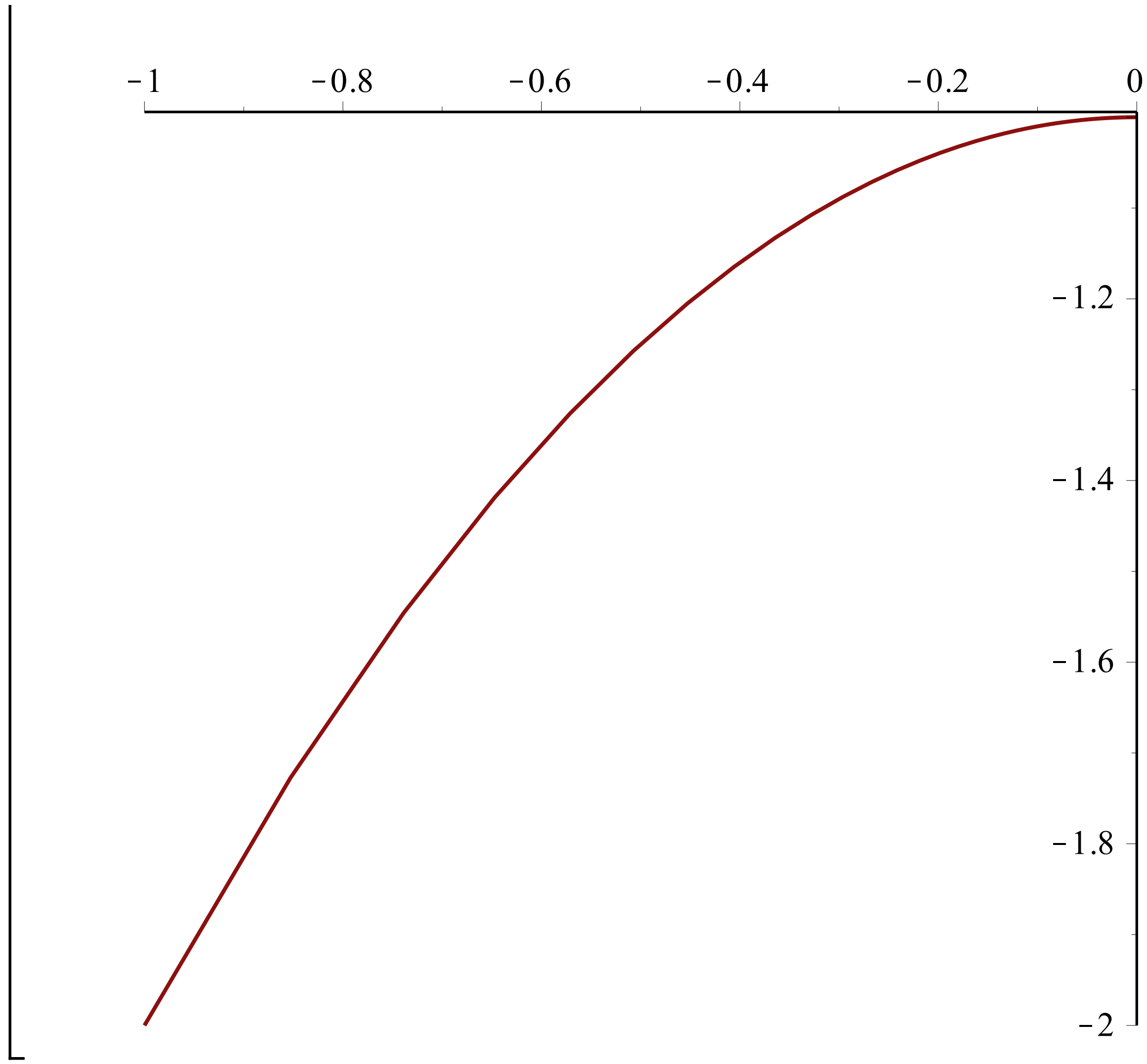
```
> IE1 := (x[k]-x[k-1])/h - x[k]*y[k] ;
  IE2 := x[k]^2 + y[k] + 1 ;
```

$$IE1 := \frac{x_k - x_{k-1}}{h} - x_k\, y_k$$

$$IE2 := x_k^2 + y_k + 1 \tag{6}$$

The step is: solve the nonlinear system IE1 with IE2

```
> STEP := proc( x0, y0, hstep )
    local RES ;
    # use x0 and x0 as guess for nonlinear system
    RES := fsolve( subs(h=hstep,x[k-1]=x0,y[k-1]=y0,{IE1,IE2}), {x[k]
  =x0,y[k]=y0} ) ;
    subs( RES, [x[k],y[k]] ) ;
  end:
> STEPS := proc( xInit, yInit, hstep, N )
    local i, XY ;
    XY := [[xInit,yInit]] ;
    for i from 1 to N do
      XY := [op(XY), STEP( XY[-1][1], XY[-1][2], hstep )] ;
    end:
    XY ;
  end:
> XY := STEPS(-1,-2,0.1,1000):
> plot(XY) ;
```



−1
−0.8
−0.6
−0.4
−0.2
0
−1.2
−1.4
−1.6
−1.8
−2