Numerical Methods for Dynamic System and Control del 7/2/2014

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1

Study the following constrained minimization problem.

$$\text{minimize} \quad f(x,y,z) = z - yz^2, \qquad \text{subject to} \qquad x = y, \qquad 0 \le y \le 1,$$

KKT system of first order condition:

$$\begin{cases} -\lambda &= 0 \\ -z^2+\lambda-\mu_1+\mu_2 &= 0 \\ -2yz+1 &= 0 \\ x-y &= 0 \\ \mu_1 y &= 0 \\ \mu_2(1-y) &= 0 \end{cases}$$

Solutions of KKT system:

$\{ x = 1, \quad y = 1, \quad z = 1/2, \quad \lambda = 0, \quad \mu_1 = 0, \quad \mu_2 = 1/4 \quad \text{SI}$

Discussion of the stationary point: $x = 1, \quad y = 1, \quad z = 1/2, \quad \lambda = 0, \quad \mu_1 = 0, \quad \mu_2 = 1/4$

$$\nabla[h, g_1] = \begin{pmatrix} 1 & -1 & 0 \\ 0 & -1 & 0 \end{pmatrix} \qquad \boldsymbol{K} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad \boldsymbol{K}^T \nabla^2 \mathcal{L} \boldsymbol{K} = [-2]$$

2

Solve the following recurrence

$$x_{k+1} = x_k + y_{k+1}, \qquad x_0 = 1$$
$$y_{k+1} = x_k + k, \qquad y_0 = 0,$$

$\mathcal{Z}-$transform:

$$(\zeta - 1)x(\zeta) - \zeta = \zeta y(\zeta),$$

$$\zeta y(\zeta) = x(\zeta) + \frac{\zeta}{(\zeta - 1)^2},$$

Solution in $\mathcal{Z}$:

$$x(\zeta) = -\frac{\zeta}{(\zeta - 1)^3},$$

$$y(\zeta) = \zeta\frac{\zeta^3 - 3\zeta^2 + 2\zeta - 1}{(\zeta - 1)^4},$$

Solution in k

$$x_k = 2^{k+1} - k - 1,$$

$$y_k = 2^k - 1,$$

3

Solve using Laplace transform the following boundary value problem

$$x''(t) + y'(t) = -\sin t$$
$$y''(t) - y(t) = -2\cos t$$
$$x(0) = 0, \quad y(0) = 1, \quad x(1) = 1, \quad y(1) = \cos(1),$$

Suggestion: Set $A = x'(0)$ and $B = y'(0)$ then using Laplace transform compute A and B which satisfy the boundary conditions.

The Laplace transform:

$$\begin{cases} s^2x(s) - A - 1 + sy(s) &= -\dfrac{1}{s^2+1} \\ s^2y(s) - B - s - y(s) &= -\dfrac{2s}{s^2+1} \end{cases}$$

Solution in s with partial fraction expansion:

$$\begin{cases} x(s) &= \dfrac{As^2 - Bs - A}{s^2(s^2-1)} &= \dfrac{A}{s^2} + \dfrac{B}{s} - \dfrac{B}{2(s+1)} - \dfrac{B}{2(s-1)} \\ y(s) &= \dfrac{Bs^2 + s^3 + B - s}{s^4 - 1} &= \dfrac{s}{s^2+1} - \dfrac{B}{2(s+1)} + \dfrac{B}{2(s-1)} \end{cases}$$

Value of A and B:

$A = 1, \qquad B = 0$

Solution in t:

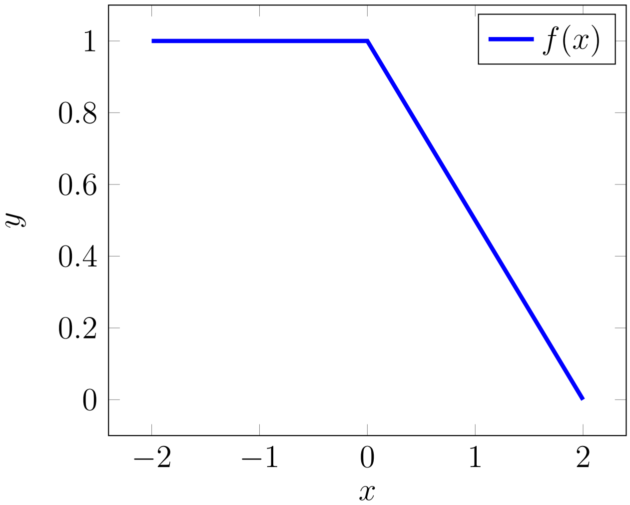
$$\begin{cases} x(t) &= t \\ y(t) &= \cos t \end{cases}$$

4

Compute the coefficients of the Fourier series of the following function:

$$f(x) = \begin{cases} 1 & \text{for } -2 \le x < 0 \\ 1 - x/2 & \text{for } 0 \le x < 2 \end{cases}$$

defined for $x \in (-2, 2)$ and extended periodically.



Coefficients a_k and b_k for the Fourier serie:

$$a_0 = \frac{3}{2}$$

$$a_k = \frac{\sin(k\pi)}{k\pi} + \frac{1}{k^2\pi^2} - \frac{\cos(k\pi)}{k^2\pi^2} = \frac{1-(-1)^k}{k^2\pi^2}$$

$$b_k = \frac{\cos(k\pi)}{k\pi} - \frac{\sin(k\pi)}{k^2\pi^2} = \frac{(-1)^k}{k\pi}$$