

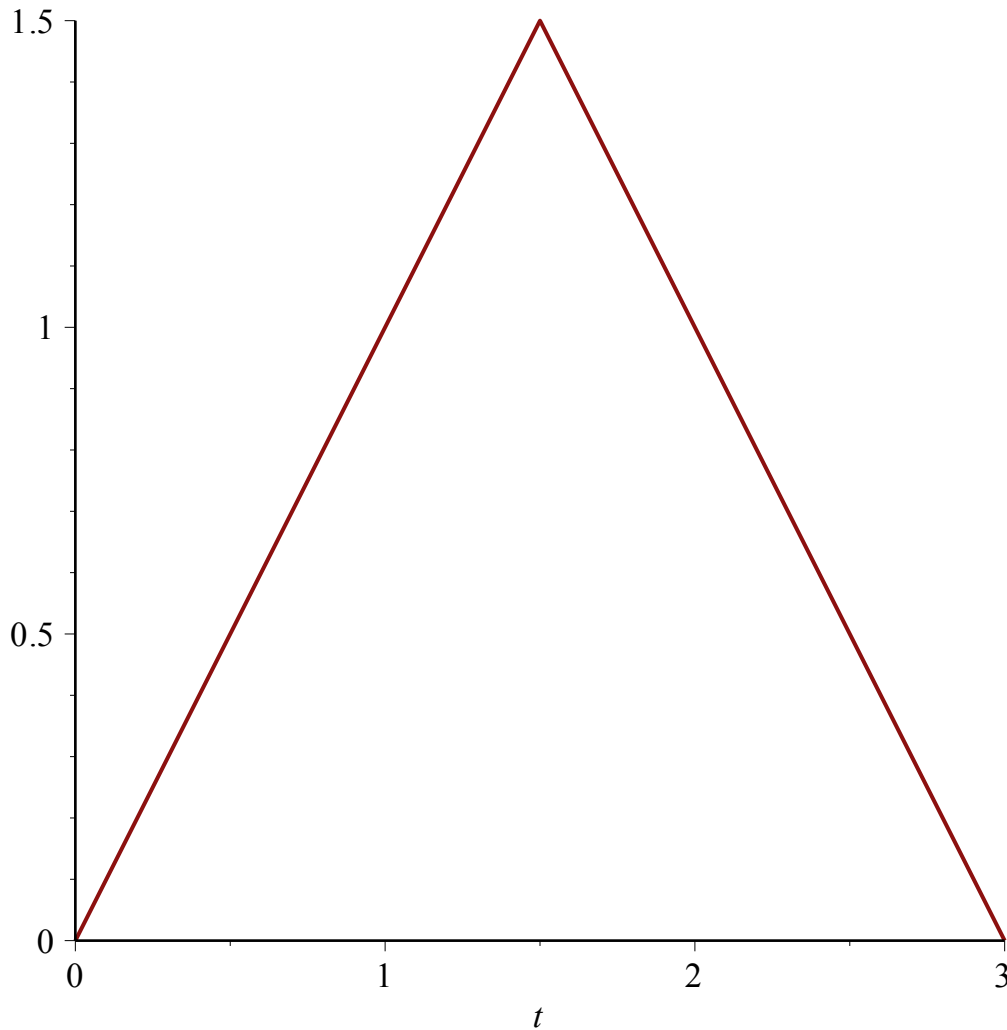
## Example of Fourier expansion (period 2\*l)

Function with period 3

```
> f := unapply( piecewise( t<3/2, t,  
                           t >= 3/2, 3-t ), t) ;  
f:= t→piecewise( t < 3/2, t, 3/2 ≤ t, 3-t )
```

(1)

```
> plot( f(t), t=0..3) ;
```



```
> ELL := 3/2 ;
```

$$ELL := \frac{3}{2}$$

(2)

```
> a0 := int( f(t), t=0..2*ELL) / ELL ;
```

$$a0 := \frac{3}{2}$$

(3)

```
> a1 := int( f(t)*cos(t*Pi/ELL), t=0..2*ELL) / ELL ;  
b1 := int( f(t)*sin(t*Pi/ELL), t=0..2*ELL) / ELL ;
```

$$a1 := -\frac{6}{\pi}$$

$$b1 := 0$$

(4)

```
> a2 := int( f(t)*cos(2*t*Pi/ELL), t=0..2*ELL) / ELL ;  
b2 := int( f(t)*sin(2*t*Pi/ELL), t=0..2*ELL) / ELL ;
```

$$a2 := 0$$

$$b2 := 0$$

(5)

```
> a3 := int( f(t)*cos(3*t*Pi/ELL), t=0..2*ELL) / ELL ;  
b3 := int( f(t)*sin(3*t*Pi/ELL), t=0..2*ELL) / ELL ;
```

$$a3 := -\frac{2}{3\pi^2}$$

$$b3 := 0$$

(6)

```
> a4 := int( f(t)*cos(4*t*Pi/ELL), t=0..2*ELL) / ELL ;  
b4 := int( f(t)*sin(4*t*Pi/ELL), t=0..2*ELL) / ELL ;
```

$$a4 := 0$$

$$b4 := 0$$

(7)

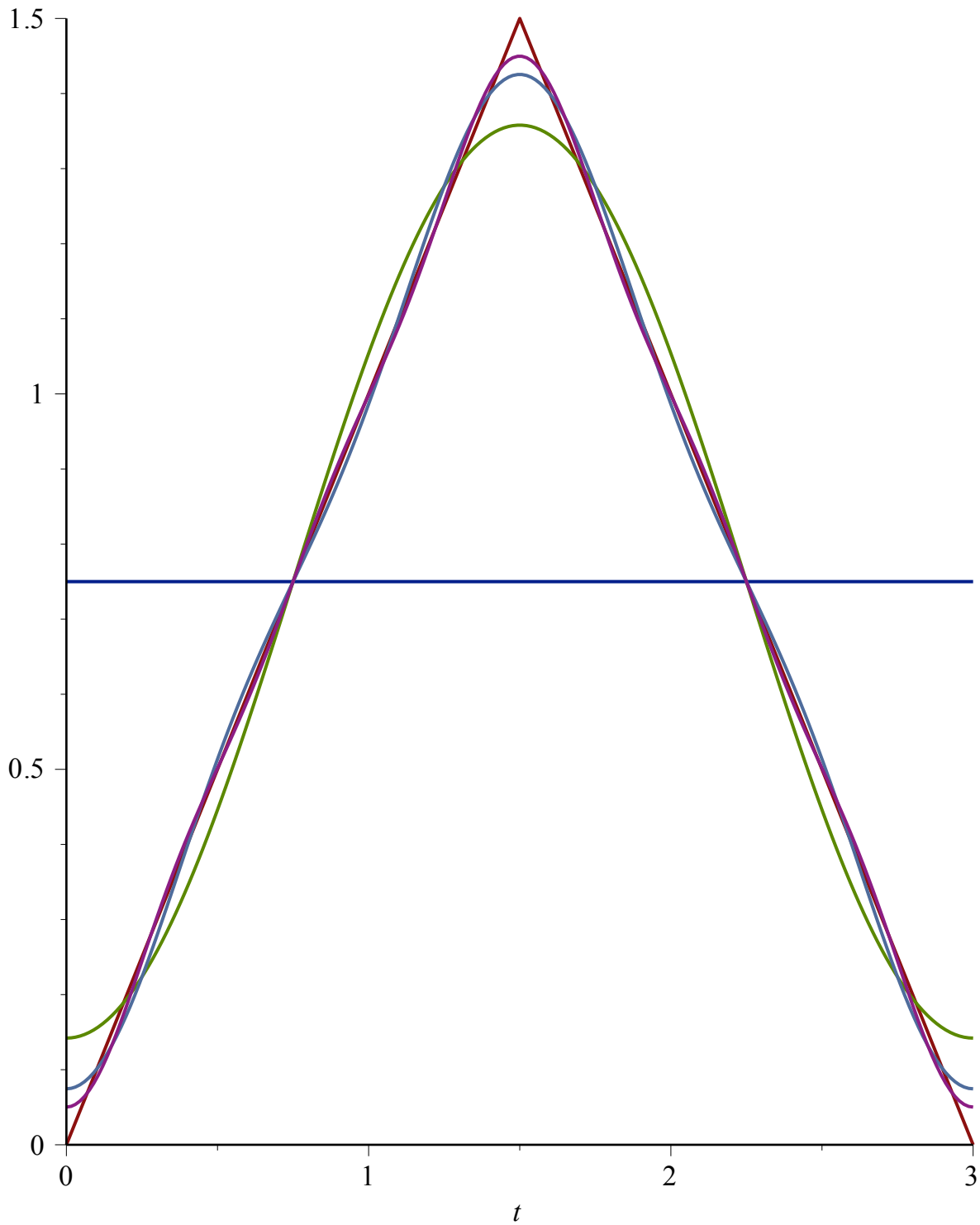
```
> a5 := int( f(t)*cos(5*t*Pi/ELL), t=0..2*ELL) / ELL ;  
b5 := int( f(t)*sin(5*t*Pi/ELL), t=0..2*ELL) / ELL ;
```

$$a5 := -\frac{6}{25\pi^2}$$

$$b5 := 0$$

(8)

```
> plot( [f(t), a0/2,  
        a0/2+a1*cos(t*Pi/ELL),  
        a0/2+a1*cos(t*Pi/ELL)+a3*cos(3*t*Pi/ELL),  
        a0/2+a1*cos(t*Pi/ELL)+a3*cos(3*t*Pi/ELL)+a5*cos(5*t*Pi/ELL)  
], t=0..2*ELL) ;
```



```
> ak := int( f(t)*cos(k*t*Pi/ELL), t=0..2*ELL) / ELL ;
```

$$ak := \frac{3}{2} \frac{\sin(k\pi) k\pi + \cos(k\pi) - 1}{k^2 \pi^2} - \frac{3}{2} \frac{\sin(k\pi) k\pi + 2 \cos(k\pi)^2 - \cos(k\pi) - 1}{k^2 \pi^2} \quad (9)$$

```
> ak := simplify(ak) assuming k::integer ;
```

(10)

$$ak := \frac{3(-1 + (-1)^k)}{k^2 \pi^2} \quad (10)$$

```
> ak := simplify(subs( k=2*q+1, ak)) assuming q::integer ;
```

$$ak := -\frac{6}{(2q+1)^2 \pi^2} \quad (11)$$

```
> Sum( ak*cos((2*q+1)*t), q=0..infinity) ;
```

$$\sum_{q=0}^{\infty} \left( -\frac{6 \cos((2q+1)t)}{(2q+1)^2 \pi^2} \right) \quad (12)$$

```
> plot( sum( ak*cos((2*q+1)*t*Pi/ELL), q=0..5 ), t=-2*ELL..2*ELL) ;
```

