COSTRUZIONE METODI MULTISTEP

$$\sum_{j=-1}^{\ell} \alpha_j y_{k-j} = h \sum_{j=-1}^{\ell} \beta_j f_{k-j} \qquad f_k = f(x_k, y_k)$$

$$\begin{cases} y' = f(x,y) \\ y(a) = y_0 \end{cases}$$

Modo 1

Usando
Formule di
quadrature

Modo 2

Usando
differenze finite
per approssimare $y'(x)$

Modo 1

$$\begin{cases} y' = f(x,y) \\ y(a) = y_a \end{cases}$$

$$y'(x) = f(x, y(x))$$

$$h = \frac{b-a}{m} \qquad x_k = a + kh$$

$$\int_{x_k}^{x_{k+1}} y'(x)\,dx = \int_{x_k}^{x_{k+1}} f(x, y(x))\,dx = \int_{x_k}^{x_{k+1}} g(x)\,dx \qquad g(x) = f(x, y(x))$$

$$\left(\frac{h^3}{12} g''(\xi) \right)$$

TRAPEZI

$$y(x_{k+1}) - y(x_k) = \frac{g(x_k) + g(x_{k+1})}{2} h + O(h^2)$$

$$= \frac{f(x_k, y(x_k)) + f(x_{k+1}, y(x_{k+1}))}{2} h + O(h^2)$$

$$y_{k+1} - y_k = \frac{h}{2} \left(f(x_k, y_k) + f(x_{k+1}, y_{k+1}) \right)$$

CRANK-NICOLSON

MODO 1 (CONT.)

$$y(x_{k+1}) - y(x_k) = \int_{x_k}^{x_{k+1}} y'(x)\,dx = \int_{x_k}^{x_{k+1}} f(x, y(x))\,dx = \int_{x_k}^{x_{k+1}} g(x)\,dx$$

$$g(x) = f(x, y(x))$$

$$\int_{x_k}^{x_{k+1}} g(x)\,dx = \int_{x_k}^{x_{k+1}} p(x)\,dx + \text{ERRORE}$$

Polinomio che interpola $g(x)$

$$g(x_k) \quad g(x_{k-1}) \quad \dots \quad g(x_{k-e})$$

$$f(x_k, y(x_k)) \quad f(x_{k-1}, y(x_{k-1})) \quad \dots \quad f(x_{k-e}, y(x_{k-e}))$$

e = INTERO = PASSI PRECEDENTI

$$y(x_{k+1}) - y(x_k) = \int_{x_k}^{x_{k+1}} p(x)\,dx + \text{ERR.}$$

COSTRUZIONE DEL POLINOMIO INTERPOLANTE

$$y(x_{n+1}) = y(x_n) + \int_{x_n}^{x_{n+1}} p(x)\,dx + \text{ERRORE}$$

$p(x)$

x	x_n	x_{n-1}	x_{n-2}	x_{n-e}
y	f_n	f_{n-1}	f_{n-2}	f_{n-e}

$$f_n = f(x_n, y(x_n)) = g(x_n)$$

$$p(x) = g[x_n] + g[x_n, x_{n-1}](x - x_n) + g[x_n, x_{n-1}, x_{n-2}](x - x_n)(x - x_{n-1}) + \dots +$$

$$g[x_n\, x_{n-1} \,..\, x_{n-e}](x - x_n)(x - x_{n-1}) \cdots (x - x_{n-e+1})$$

polinomio interp. formule diff. divise di Newton

$$P(x) = g[x_k] +$$
$$g[x_k, x_{k-1}](x - x_k) +$$
$$g[x_k, x_{k-1}, x_{k-2}](x - x_k)(x - x_{k-1}) + \cdots +$$

$$g[x_k x_{k-1} \ldots x_{k-e}](x - x_k)(x - x_{k-1}) \cdots (x - x_{k-e+1})$$

$$\int_{x_k}^{x_{k+1}} p(x)\,dx = g[x_k] \int_{x_k}^{x_{k+1}} 1\,dx + g[x_k, x_{k-1}] \int_{x_k}^{x_{k+1}} (x - x_k)\,dx$$

$$+ g[x_k, x_{k-1}, x_{k-2}] \int_{x_k}^{x_{k+1}} (x - x_k)(x - x_{k-1})\,dx +$$

$$+ g[x_k x_{k-1} \quad x_{k-e}] \int_{x_k}^{x_{k+1}} (x - x_k)(x - x_{k-1}) \cdots (x - x_{k-e+1})\,dx$$

bunları ×h

$$\int_{x_n}^{x_{n+1}} 1\,dx = \int_{x_n}^{x_n+l_1} 1\,dx = \int_0^{l_1} 1\,dz = l_1 \qquad x = x_n + z$$

$$x - x_{n-1}$$
$$= x - (x_n - l_1)$$

$$\int_{x_n}^{x_{n+1}} x - x_n\,dx = \int_0^{l_1} z\,dz = \frac{l_1^2}{2}$$

$$\int_{x_n}^{x_{n+1}} (x - x_n)(x - x_{n-1})\,dx = \int_0^{l_1} z(z + l_1)\,dz = \int_0^{l_1} z^2 + l_1 z\,dz$$

$$= \left[\frac{z^3}{3} + l_1 \frac{z^2}{2}\right]_0^{l_1} = \frac{l_1^3}{3} + \frac{l_1^3}{2} = \frac{5}{6} l_1^3$$

$$\int_{x_n}^{x_{n+1}} (x - x_n)(x - x_{n-1}) \cdots (x - x_{n-r})\,dx = (\text{numero})\, l_1^{r+2}$$

$g[x_n]$

$$\frac{g[x_n]-g[x_{n-1}]}{(x_n - x_{n-1}) = h} = g[x_n, x_{n-1}]$$

$g[x_{n-1}]$

$$\frac{g[x_{n-1}]-g[x_{n-2}]}{(x_{n-1} - x_{n-2}) = h} = g[x_{n-1}, x_{n-2}]$$

$g[x_{n-2}]$

$$\frac{g[x_{n-2}]-g[x_{n-3}]}{(x_{n-2} - x_{n-3}) = h} = g[x_{n-2}, x_{n-3}]$$

$g[x_{n-3}]$

$$\frac{g[x_n; x_{n-1}]-g[x_{n-1}\ x_{n-2}]}{(x_n - x_{n-2}) = 2h} = g[x_n, x_{n-1}, x_{n-2}]$$

MODO 1 (CONT.)

$$y(x_{n+1}) - y(x_n) = \int_{x_n}^{x_{n+1}} p(x)\,dx + \text{ERRORE}$$

$$= h\,g(x_n) + \frac{h^2}{2} g[x_n, x_{n-1}]$$

$$+ c_j h^{j+1} g[x_n x_{n-1} \dots x_{n-j}] +$$

$$+ \dots + c_e h^{e+1} g[x_n x_{n-1} \dots x_{n-e}]$$

$$h^{j+1} g[x_n x_{n-1} \dots x_{n-j}] = h\,(a\,g(x_n) + b\,g(x_{n-1})$$
$$+ d\,g(x_{n-2}) \dots)$$

DIFFERENZE NON DIVISE

$g\{x\} = g(x)$ $\qquad g\{x, y\} = g(x) - g(y)$

$$g\{x, -, y\} = g\{x, -\} - g\{-, y\}$$

$$\begin{array}{l} g(x_0) \\ \quad \searrow g(x_0) - g(x_1) = g\{x_0\, x_1\} \\ g(x_1) \\ \quad \searrow g(x_1) - g(x_2) = g\{x_1\, x_1\} \\ g(x_2) \\ \quad \searrow g(x_2) - g(x_3) = g\{x_2\, x_3\} \\ g(x_3) \end{array} \qquad \begin{array}{l} g\{x_0\, x_1\} - g\{x_1\, x_3\} \\ g\{x_1\, x_2\} - g\{x_2\, x_3\} \end{array}$$

$$y(x_{n+1}) - y(x_n) = h\, g(x_n) + \frac{h}{2} g\{x_k, x_{k-1}\} + \frac{5}{3} h\, g\{x_n\, x_{k-1}\, x_{n-2}\}$$

$$+ \dots + c_e\, h\, g\{x_n\, x_{n-1} \ \ x_{n-e}\}$$

$+$ ERRORE

STESSA COSA USANDO LAGRANGE

$$y(x_{n+1}) - y(x_n) = \int_{x_n}^{x_{n+1}} p(x)\,dx + \text{ERRORE}$$

$$p(x) = g(x_n) L_0(x) + g(x_{n-1}) L_1(x) + g(x_{n-2}) L_2(x)$$

$$+ \cdots + g(x_{n-e}) L_e(x)$$

$$L_0(x) = \begin{cases} 1 & \text{se } x = x_n \\ 0 & \text{se } x = x_{n-j} \end{cases} \qquad L_i(x) = \begin{cases} 1 & \text{se } x = x_{n-i} \\ 0 & x = x_{n-j} \\ & j \neq i \end{cases}$$

$$\int_{x_n}^{x_{n+1}} p(x)\,dx = g(x_n) \int_{x_n}^{x_{n+1}} L_0(x) + g(x_{n-1}) \int_{x_n}^{x_{n+1}} L_1(x)\,dx$$

$$+ \cdots +$$

STESSA COSA USANDO LAGRANGE

$$p(x) = g(x_n) L_0(x) + g(x_{n-1}) L_1(x) + g(x_{n-2}) L_2(x) + \dots + g(x_{n-e}) L_e(x)$$

$$\int_{x_n}^{x_{n+1}} p(x) = g(x_n) c_0 + g(x_{n-1}) c_1 + g(x_{n-2}) c_2 + \dots + g(x_{n-e}) c_e$$

$$c_j = \int_{x_n}^{x_{n+1}} L_j(x)\,dx = h \text{ (NUMERO)}$$

$$y(x_{n+1}) - y(x_n) = h \sum c_j \underbrace{g(x_{n-e})}_{f(x_{n-e},\, y(x_{n-e}))} + \text{ERRORE}$$

$$y(x_{n+1}) - y(x_n) = h \sum c_j \underbrace{g(x_{n-j})}_{f(x_{n-j},\, y(x_{n-j}))} + \text{ERRORE}$$

TRASCURANDO ERRORE

$$\boxed{y_{n+1} = y_n + h \sum c_j f(x_{n-j}, y_{n-j})}$$

CASO PARTICOLARE DI METODO MULTI STEP

METODI ADAMS-BASHFORTH

$$y'(x) = f(x, y(x)) \qquad \int_{x_n}^{x_{n+1}} y'(x)\,dx \int_{x_n}^{x_{n+1}} f(x, y(x))\,dx$$

polinomio interpolante

$$f(x_{n-j}, y(x_{n-j})) \quad j = 0, 1, \dots, e$$

USATO PER APPROSSIMARE INTEGRALE

METODI ADAMS-MOULTON

$$\int_{x_n}^{x_{n+1}} y'(x)\,dx = \int_{x_n}^{x_{n+1}} f(x, y(x))\,dx$$ Polinomio interp.

$$f(x_{n-j}, y(x_{n-j})) \quad j = -1, 0, \dots, e$$

Metodo implicito

COME COMBINARE AM (implicito)
AB (esplicito)

AM metodo implicito ($\alpha_{-1} = 1$) — CORRECTOR

$$y_{n+1} + \sum_{j=0}^{\ell} \alpha_j y_{n-j} = h \beta_{-1} f(x_{n+1}, y_{n+1}) + h \sum_{j=0}^{\ell} \beta_j f(x_{n-j}, y_{n-j})$$

AB metodo esplicito ($\alpha_{-1} = 1$) — PREDICTOR

$$\tilde{y}_{n+1} + \sum_{j=0}^{\ell} \tilde{\alpha}_j y_{n-j} = h \sum_{j=0}^{\ell} \tilde{\beta}_j f(x_{n-j}, y_{n-j})$$

$\tilde{y}_{n+1}$

Metodi predictor corrector

AB esplicito come PREDICTOR

$$\tilde{y}_{k+1} + \sum_{j=0}^{e} \alpha_j y_{k-j} = h \sum_{j=0}^{e} \beta_j f_{k-j}$$

AM implicito come CORRECTOR ⇒ (esplicito)

$$y_{k+1} + \sum_{j=0}^{e} \alpha_j y_{k-j} = h \beta_{-1} f(x_{k+1}, \tilde{y}_{k+1}) + h \sum_{j=0}^{e} \beta_j f(x_{k-j}, y_{k-j})$$

Ποδα 2

$$y'(x_n) = f(x_n, y(x_n))$$

$$y'(x_n) = \frac{y(x_{n+1}) - y(x_n)}{h} + O(h)$$

$$\frac{y(x_{n+1}) - y(x_n)}{h} + O(h) = f(x_n, y(x_n))$$

$$y'(x_n) = \frac{3y(x_n) - 4y(x_{n-1}) + y(x_{n-2})}{2h} + O(h^2)$$

$$3y(x_n) - 4y(x_{n-1}) + y(x_{n-2}) = 2h\, f(x_n, y(x_n)) + O(h^4)$$

$y'(x_{k+1}) = f(x_{k+1}, y_{k+1})$

↖ differenze finite su $y(x_{k+1}), y(x_k), y(x_{k-1})$ ..

$\Rightarrow$ BDF

BACKWARD Differential Formulae

$$\sum_{j=-1}^{r} \alpha_j y_{k-j} = \beta h f_{k+1} = h f(x_{k+1}, y_{k+1})$$