ESERCIZIO CON R.K. IMPLICITO

$$\begin{cases} y' = x^2 + y \\ y(0) = 1 \end{cases}$$

TABLEAU

$$\begin{array}{c|cc} \frac{1}{2} - \frac{\sqrt{3}}{6} & \frac{1}{4} & \frac{1}{4} - \frac{\sqrt{3}}{6} \\ \frac{1}{2} + \frac{\sqrt{3}}{6} & \frac{1}{4} + \frac{\sqrt{3}}{6} & \frac{1}{4} \\ \hline & \frac{1}{2} & \frac{1}{2} \end{array}$$

c A b

RK (GENERICO)

$$y_{k+1} = y_k + \frac{1}{2} K_1 + \frac{1}{2} K_2$$

$$K_1 = \boxed{h f\left(x_k + \left(\frac{1}{2} - \frac{\sqrt{3}}{6}\right) h, y_k + \frac{1}{4} K_1 + \left(\frac{1}{4} - \frac{\sqrt{3}}{6}\right) K_2\right)}$$

BRUTTA

$$= h\left(x_k + \left(\frac{1}{2} - \frac{\sqrt{3}}{6}\right) h\right)^2 + h\left(y_k + \frac{1}{4} K_1 + \left(\frac{1}{4} - \frac{\sqrt{3}}{6}\right) K_2\right)$$

$$K_2 = \boxed{h f\left(x_k + \left(\frac{1}{2} + \frac{\sqrt{3}}{6}\right) h, y_k + \left(\frac{1}{4} + \frac{\sqrt{3}}{6}\right) K_1 + \frac{1}{4} K_2\right)}$$

$$= h\left(x_k + \left(\frac{1}{2} + \frac{\sqrt{3}}{6}\right) h\right)^2 + h\left(y_k + \left(\frac{1}{4} + \frac{\sqrt{3}}{6}\right) K_1 + \frac{1}{4} K_2\right)$$

CALCOLO ORDINE

$$
\begin{array}{c|cc}
\frac{1}{2}-\frac{\sqrt{3}}{6} & \frac{1}{4} & \frac{1}{4}-\frac{\sqrt{3}}{6} \\
\frac{1}{2}+\frac{\sqrt{3}}{6} & \frac{1}{4}+\frac{\sqrt{3}}{6} & \frac{1}{4} \\
\hline
 & \frac{1}{2} & \frac{1}{2}
\end{array}
$$

- **CONSISTENZA**

$$\frac{1}{4}+\left(\frac{1}{4}-\frac{\sqrt{3}}{6}\right) = \frac{1}{2}-\frac{\sqrt{3}}{6} = c_1 \quad (\text{ok})$$

$$\frac{1}{4}+\frac{\sqrt{3}}{6}+\frac{1}{4} = \frac{1}{2}+\frac{\sqrt{3}}{6} = c_2 \quad (\text{ok})$$

- **ORDINE 1**

$$\frac{1}{2}+\frac{1}{2} = 1 \quad (\text{ok})$$

- **ORDINE 2**

$$\begin{pmatrix} \frac{1}{2}-\frac{\sqrt{3}}{6} \\ \frac{1}{2}+\frac{\sqrt{3}}{6} \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} = \frac{1}{2} \quad (\text{ok})$$

- **ORDINE 3** $\sum bc^2 = \frac{1}{3}$

$$c^2 = \begin{pmatrix} \frac{1}{4}+\frac{3}{36}-\frac{\sqrt{3}}{6} \\ \frac{1}{4}+\frac{3}{36}+\frac{\sqrt{3}}{6} \end{pmatrix} = \begin{pmatrix} \frac{1}{3}-\frac{\sqrt{3}}{6} \\ \frac{1}{3}+\frac{\sqrt{3}}{6} \end{pmatrix}$$

$$bc^2 = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{3}-\frac{\sqrt{3}}{6} \\ \frac{1}{3}+\frac{\sqrt{3}}{6} \end{pmatrix} = \frac{1}{3} \quad (\text{ok})$$

$$b^T A c = \frac{1}{6} \quad (\text{ok})$$

$$Ac = \begin{pmatrix} \frac{1}{4} & \frac{1}{4}-\frac{\sqrt{3}}{6} \\ \frac{1}{4}+\frac{\sqrt{3}}{6} & \frac{1}{4} \end{pmatrix} \begin{pmatrix} \frac{1}{2}-\frac{\sqrt{3}}{6} \\ \frac{1}{2}+\frac{\sqrt{3}}{6} \end{pmatrix} = \begin{pmatrix} \frac{1}{4}-\frac{\sqrt{3}}{12}-\frac{3}{36} \\ \frac{1}{4}+\frac{\sqrt{3}}{12}-\frac{3}{36} \end{pmatrix} = \begin{pmatrix} \frac{1}{6}-\frac{\sqrt{3}}{12} \\ \frac{1}{6}+\frac{\sqrt{3}}{12} \end{pmatrix}$$

ORDINE 4

$$Ac = \begin{pmatrix} \frac{1}{6} - \frac{\sqrt{3}}{12} \\ \frac{1}{6} + \frac{\sqrt{3}}{12} \end{pmatrix} \qquad c^2 = \begin{pmatrix} \frac{1}{3} - \frac{\sqrt{3}}{6} \\ \frac{1}{3} + \frac{\sqrt{3}}{6} \end{pmatrix}$$

$$\begin{array}{c|cc} \frac{1}{2} - \frac{\sqrt{3}}{6} & \frac{1}{4} & \frac{1}{4} - \frac{\sqrt{3}}{6} \\ \frac{1}{2} + \frac{\sqrt{3}}{6} & \frac{1}{4} + \frac{\sqrt{3}}{6} & \frac{1}{4} \\ \hline & \frac{1}{2} & \frac{1}{2} \end{array}$$

$$c^3 = \begin{pmatrix} \left(\frac{1}{3} - \frac{\sqrt{3}}{6}\right)\left(\frac{1}{2} - \frac{\sqrt{3}}{6}\right) \\ \left(\frac{1}{3} + \frac{\sqrt{3}}{6}\right)\left(\frac{1}{2} + \frac{\sqrt{3}}{6}\right) \end{pmatrix} = \begin{pmatrix} \frac{1}{6} + \frac{3}{36} - \frac{5}{36}\sqrt{3} \\ \frac{1}{6} + \frac{3}{36} + \frac{5}{36}\sqrt{3} \end{pmatrix} = \begin{pmatrix} \frac{1}{4} - \frac{5}{36}\sqrt{3} \\ \frac{1}{4} + \frac{5}{36}\sqrt{3} \end{pmatrix}$$

$$b^T A = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} \frac{1}{4} & \frac{1}{4} - \frac{\sqrt{3}}{6} \\ \frac{1}{4} + \frac{\sqrt{3}}{6} & \frac{1}{4} \end{pmatrix} = \begin{pmatrix} \frac{1}{4} + \frac{\sqrt{3}}{12} & \frac{1}{4} - \frac{\sqrt{3}}{12} \end{pmatrix}$$

$$\sum b c^3 = \frac{1}{4} \qquad \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{4} - \frac{5}{36}\sqrt{3} \\ \frac{1}{4} + \frac{5}{36}\sqrt{3} \end{pmatrix} = \frac{1}{4} \text{ (ok)}$$

$$bc = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} \otimes \begin{pmatrix} \frac{1}{2} - \frac{\sqrt{3}}{6} \\ \frac{1}{2} + \frac{\sqrt{3}}{6} \end{pmatrix} = \begin{pmatrix} \frac{1}{4} - \frac{\sqrt{3}}{12} \\ \frac{1}{4} + \frac{\sqrt{3}}{12} \end{pmatrix}$$

$$(bc)(Ac) = \begin{pmatrix} \frac{1}{4} - \frac{\sqrt{3}}{12} \\ \frac{1}{4} + \frac{\sqrt{3}}{12} \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{6} - \frac{\sqrt{3}}{12} \\ \frac{1}{6} + \frac{\sqrt{3}}{12} \end{pmatrix}$$

$$= \frac{1}{24} + \overset{\frac{1}{48}}{\frac{3}{144}} - \cancel{\frac{\sqrt{3}}{144}} 5 + \frac{1}{24} + \frac{3}{144} + \cancel{\frac{\sqrt{3}}{144}} 5$$

$$\frac{1}{12} + \frac{1}{24} = \frac{1}{8} = \text{(ok)}$$

ORDINE 4 (continua)

$$c^2 = \begin{pmatrix} \frac{1}{3} - \frac{\sqrt{3}}{6} \\ \frac{1}{3} + \frac{\sqrt{3}}{6} \end{pmatrix}$$

$b^T A c^2$

$$\begin{pmatrix} \frac{1}{4} & \frac{1}{4} - \frac{\sqrt{3}}{6} \\ \frac{1}{4} + \frac{\sqrt{3}}{6} & \frac{1}{4} \end{pmatrix} \begin{pmatrix} \frac{1}{3} - \frac{\sqrt{3}}{6} \\ \frac{1}{3} + \frac{\sqrt{3}}{6} \end{pmatrix} = \begin{pmatrix} \frac{1}{6} - \frac{\sqrt{3}}{18} - \frac{3}{36} \\ \frac{1}{6} + \frac{\sqrt{3}}{18} - \frac{3}{36} \end{pmatrix} = \begin{pmatrix} \frac{1}{12} - \frac{\sqrt{3}}{18} \\ \frac{1}{12} + \frac{\sqrt{3}}{18} \end{pmatrix}$$

$$b^T A c^2 = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} \frac{1}{12} - \frac{\sqrt{3}}{18} \\ \frac{1}{12} + \frac{\sqrt{3}}{18} \end{pmatrix} = \frac{1}{12} \text{ (OK)}$$

$$Ac = \begin{pmatrix} \frac{1}{6} - \frac{\sqrt{3}}{12} \\ \frac{1}{6} + \frac{\sqrt{3}}{12} \end{pmatrix}$$

$$b^T A = \begin{pmatrix} \frac{1}{4} + \frac{\sqrt{3}}{12} & \frac{1}{4} - \frac{\sqrt{3}}{12} \end{pmatrix}$$

$$b^T A A c = \begin{pmatrix} \frac{1}{4} + \frac{\sqrt{3}}{12}, & \frac{1}{4} - \frac{\sqrt{3}}{12} \end{pmatrix} \begin{pmatrix} \frac{1}{6} - \frac{\sqrt{3}}{12} \\ \frac{1}{6} + \frac{\sqrt{3}}{12} \end{pmatrix}$$

$$= \frac{1}{12} - \frac{3}{144} - \frac{3}{144} = \frac{1}{12} - \frac{1}{24} = \frac{1}{24} \text{ (OK)}$$

FARE UN PASSO CON $h = \frac{1}{2}$

$Y_{k+1} = Y_k + \frac{1}{2} K_1 + \frac{1}{2} K_2$ $\qquad k=0 \quad x_k = 0$

$$K_1 = h\left(x_k + \left(\frac{1}{2} - \frac{\sqrt{3}}{6}\right)h\right)^2 + h\left(Y_k + \frac{1}{4}K_1 + \left(\frac{1}{4} - \frac{\sqrt{3}}{6}\right)K_2\right)$$

$$K_2 = h\left(x_k + \left(\frac{1}{2} + \frac{\sqrt{3}}{6}\right)h\right)^2 + h\left(Y_K + \left(\frac{1}{4} + \frac{\sqrt{3}}{6}\right)K_1 + \frac{1}{4}K_2\right)$$

$$K_1 = \frac{1}{2}\left(\left(\frac{1}{2} - \frac{\sqrt{3}}{6}\right)\frac{1}{2}\right)^2 + \frac{1}{2}\left(1 + \frac{1}{4}K_1 + \left(\frac{1}{4} - \frac{\sqrt{3}}{6}\right)K_2\right)$$

$$K_2 = \frac{1}{2}\left(\left(\frac{1}{2} + \frac{\sqrt{3}}{6}\right)\frac{1}{2}\right)^2 + \frac{1}{2}\left(1 + \left(\frac{1}{4} + \frac{\sqrt{3}}{6}\right)K_1 + \frac{1}{4}K_2\right)$$

$$K_1 = \frac{1}{8}\left(\frac{1}{4} + \frac{3}{36} - \frac{\sqrt{3}}{6}\right) + \frac{1}{2} + \frac{1}{8}K_1 + \left(\frac{1}{8} - \frac{\sqrt{3}}{12}\right)K_2$$

$$K_2 = \frac{1}{8}\left(\frac{1}{4} + \frac{3}{36} + \frac{\sqrt{3}}{6}\right) + \frac{1}{2} + \left(\frac{1}{8} + \frac{\sqrt{3}}{12}\right)K_1 + \frac{1}{4}K_2$$

$$K_1 = \frac{1}{8}\left(\frac{1}{4} + \frac{3}{36} - \frac{\sqrt{3}}{6}\right) + \frac{1}{2} + \frac{1}{8}K_1 + \left(\frac{1}{8} - \frac{\sqrt{3}}{12}\right)K_2$$

$$K_2 = \frac{1}{8}\left(\underline{\frac{1}{4} + \frac{3}{36}} + \frac{\sqrt{3}}{6}\right) + \frac{1}{2} + \left(\frac{1}{8} + \frac{\sqrt{3}}{12}\right)K_1 + \frac{1}{8}K_2$$

$$\begin{cases} \frac{7}{8}K_1 + \left(\frac{\sqrt{3}}{12} - \frac{1}{8}\right)K_2 = \frac{1}{2} + \frac{1}{8}\left(\frac{1}{3} - \frac{\sqrt{3}}{6}\right) \\ -\left(\frac{1}{8} + \frac{\sqrt{3}}{12}\right)K_1 + \frac{7}{8}K_2 = \frac{1}{2} + \frac{1}{8}\left(\frac{1}{3} + \frac{\sqrt{3}}{6}\right) \end{cases}$$

$$\Rightarrow \quad K_1 = 0{,}559\ldots \qquad K_2 = 0{,}832\ldots$$

$$Y_1 = Y_0 + \frac{1}{2}K_1 + \frac{1}{2}K_2$$

$$= 1 + \frac{1}{2}0{,}559 + \frac{1}{2}0{,}832$$

$$= \underline{1{,}696\ldots}$$