

> restart;

> with(inttrans):

Sistema di tre equazioni differenziali del secondo ordine

> eq1 := (D@@2)(x)(t)+(D@@2)(y)(t)+(D@@2)(z)(t)=1;

$$eq1 := D^{(2)}(x)(t) + D^{(2)}(y)(t) + D^{(2)}(z)(t) = 1$$

> eq2 := (D@@2)(x)(t)-3*(D@@2)(y)(t)-5*(D@@2)(z)(t)=exp(t);

$$eq2 := D^{(2)}(x)(t) - 3 D^{(2)}(y)(t) - 5 D^{(2)}(z)(t) = e^t$$

> eq3 :=

(D@@2)(x)(t)-D(x)(t)-(D@@2)(y)(t)-(D@@2)(z)(t)+3*D(z)(t)=exp(t)*sin(3*t);

$$eq3 := D^{(2)}(x)(t) - D(x)(t) - D^{(2)}(y)(t) - D^{(2)}(z)(t) + 3 D(z)(t) = e^t \sin(3 t)$$

Condizioni Iniziali

> ini := [x(0)=1,y(0)=-1,z(0)=1,D(x)(0)=0,D(y)(0)=-10,D(z)(0)=1];

$$ini := [x(0) = 1, y(0) = -1, z(0) = 1, D(x)(0) = 0, D(y)(0) = -10, D(z)(0) = 1]$$

Cambio di variabile per migliorare la leggibilità

> SUBS :=

{laplace(x(t),t,s)=x(s),laplace(y(t),t,s)=y(s),laplace(z(t),t,s)=z(s)};

$$SUBS := \{laplace(y(t), t, s) = y(s), laplace(z(t), t, s) = z(s), laplace(x(t), t, s) = x(s)\}$$

Sistema Trasformato

> eq1T := subs(SUBS,laplace(eq1,t,s));

eq2T := subs(SUBS,laplace(eq2,t,s));

eq3T := subs(SUBS,laplace(eq3,t,s));

$$eq1T := s^2 x(s) - D(x)(0) - s x(0) + s^2 y(s) - D(y)(0) - s y(0) + s^2 z(s) - D(z)(0) - s z(0) = \frac{1}{s}$$

$$eq2T := s^2 x(s) - D(x)(0) - s x(0) - 3 s^2 y(s) + 3 D(y)(0) + 3 s y(0) - 5 s^2 z(s) + 5 D(z)(0) + 5 s z(0) = \frac{1}{s-1}$$

$$eq3T := s^2 x(s) - D(x)(0) - s x(0) - s x(s) + x(0) - s^2 y(s) + D(y)(0) + s y(0) - s^2 z(s) + D(z)(0) + s z(0) + 3 s z(s) - 3 z(0) = \frac{3}{s^2 - 2 s + 10}$$

Sostituzione delle condizioni al contorno

> eq1TT := subs(ini,eq1T);

eq2TT := subs(ini,eq2T);

eq3TT := subs(ini,eq3T);

$$eq1TT := s^2 x(s) + 9 - s + s^2 y(s) + s^2 z(s) = \frac{1}{s}$$

$$eq2TT := s^2 x(s) - 25 + s - 3 s^2 y(s) - 5 s^2 z(s) = \frac{1}{s-1}$$

$$eq3TT := s^2 x(s) - 11 - s - s x(s) - s^2 y(s) - s^2 z(s) + 3 s z(s) = \frac{3}{s^2 - 2 s + 10}$$

Ho ottenuto un sistema algebrico equivalente.

Risoluzione del sistema lineare rispetto a x(s) e y(s)

> SOL := solve({eq1TT, eq2TT, eq3TT}, {x(s), y(s), z(s)});

$$SOL := \left\{ \begin{array}{l} x(s) = \frac{14 s^4 + 110 s^3 + 4 s^6 - 2 s^5 - 187 s^2 + 178 s - 90}{2 s^3 (2 s^4 - s^3 + 9 s^2 + 40 s - 50)}, \\ y(s) = -\frac{-13 s^4 + 177 s^3 + 2 s^6 + 192 s^2 - 358 s - 10 + 19 s^5}{s^3 (2 s^4 - s^3 + 9 s^2 + 40 s - 50)}, \\ z(s) = \frac{4 s^6 + 2 s^5 + 160 s^3 - 231 s^2 + 86 s - 30}{2 s^3 (2 s^4 - s^3 + 9 s^2 + 40 s - 50)} \end{array} \right\}$$

Sostituzione per migliorare la leggibilità

**> SUBSINV := {invlaplace(x(s), s, t)=x(t),
invlaplace(y(s), s, t)=y(t),
invlaplace(z(s), s, t)=z(t)};**

SUBSINV := {invlaplace(x(s), s, t) = x(t), invlaplace(y(s), s, t) = y(t), invlaplace(z(s), s, t) = z(t)}

> subs(SUBSINV, invlaplace(SOL, s, t)) ;

$$\left\{ \begin{array}{l} y(t) = -\frac{513}{250} + \frac{16356}{14875} e^{\left(-\frac{5}{2}t\right)} - \frac{183}{25} t - \frac{1}{10} t^2 + \frac{1}{5950} e^t (567 \cos(3 t) + 231 \sin(3 t) - 850), \\ z(t) = \frac{467}{250} - \frac{10904}{14875} e^{\left(-\frac{5}{2}t\right)} - \frac{31}{50} t + \frac{3}{20} t^2 - \frac{1}{5950} e^t (378 \cos(3 t) + 154 \sin(3 t) + 425), \\ x(t) = \frac{148}{125} - \frac{5452}{14875} e^{\left(-\frac{5}{2}t\right)} - \frac{53}{50} t + \frac{9}{20} t^2 + \frac{1}{5950} (-189 \cos(3 t) - 77 \sin(3 t) + 1275) e^t \end{array} \right\}$$

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