Pendulum in cartesian coordinates

Taylor based numerical scheme with Baumgarte stabilization

```
> restart:
```

Pendulum equation

```
> EQ1 := mass*diff(x(t),t,t)+2*x(t)*lambda(t) ;
  EQ2 := mass*diff(y(t),t,t)+2*y(t)*lambda(t)+mass*g ;
  EQ3 := x(t)^2+y(t)^2-1 ;
```

$$EQ1 := mass\left(\frac{d^2}{dt^2}x(t)\right)+2\,x(t)\,\lambda(t)$$

$$EQ2 := mass\left(\frac{d^2}{dt^2}y(t)\right)+2\,y(t)\,\lambda(t)+mass\,g$$

$$EQ3 := x(t)^2+y(t)^2-1 \tag{1}$$

Derivate constraint two times

```
> DEQ3  := diff(EQ3,t);
  DDEQ3 := diff(DEQ3,t);
```

$$DEQ3 := 2\,x(t)\left(\frac{d}{dt}x(t)\right)+2\,y(t)\left(\frac{d}{dt}y(t)\right)$$

$$DDEQ3 := 2\left(\frac{d}{dt}x(t)\right)^2+2\,x(t)\left(\frac{d^2}{dt^2}x(t)\right)+2\left(\frac{d}{dt}y(t)\right)^2+2\,y(t)\left(\frac{d^2}{dt^2}y(t)\right) \tag{2}$$

Substitute DDEQ3 with stabilized equation

```
> SEQ3 := DDEQ3 + 2*zeta*omega*DEQ3 + omega^2*EQ3 ;
```

$$SEQ3 := 2\left(2\,x(t)\left(\frac{d}{dt}x(t)\right)+2\,y(t)\left(\frac{d}{dt}y(t)\right)\right)\omega\,\zeta+\left(x(t)^2+y(t)^2-1\right)\omega^2$$
$$+2\left(\frac{d}{dt}x(t)\right)^2+2\,x(t)\left(\frac{d^2}{dt^2}x(t)\right)+2\left(\frac{d}{dt}y(t)\right)^2+2\,y(t)\left(\frac{d^2}{dt^2}y(t)\right) \tag{3}$$

Solve for second derivative

```
> RESACC := solve( {EQ1,EQ2}, diff({x(t),y(t)},t,t) ) ;
```

$$RESACC := \left\{\frac{d^2}{dt^2}x(t) = -\frac{2\,x(t)\,\lambda(t)}{mass},\ \frac{d^2}{dt^2}y(t) = -\frac{2\,y(t)\,\lambda(t)+mass\,g}{mass}\right\} \tag{4}$$

Solve for multiplier

```
> RESLAMBDA := solve( subs(RESACC,SEQ3), {lambda(t)} ) ;
```

$$RESLAMBDA := \left\{\lambda(t) = \frac{1}{4}\,\frac{1}{x(t)^2+y(t)^2}\left(mass\left(x(t)^2\,\omega^2+4\,x(t)\left(\frac{d}{dt}x(t)\right)\omega\,\zeta\right.\right.\right.$$
$$+y(t)^2\,\omega^2+4\,y(t)\left(\frac{d}{dt}y(t)\right)\omega\,\zeta-2\,y(t)\,g+2\left(\frac{d}{dt}x(t)\right)^2+2\left(\frac{d}{dt}y(t)\right)^2-\omega^2$$
$$\left.\left.\left.\right)\right)\right\} \tag{5}$$

Change names

```
> SUBS := { diff(x(t),t,t) = ax(t),
            diff(y(t),t,t) = ay(t),
            diff(x(t),t)   = u(t),
            diff(y(t),t)   = v(t) } ;
```

$$SUBS := \left\{ \frac{d}{dt}\, x(t) = u(t),\ \frac{d^2}{dt^2}\, x(t) = ax(t),\ \frac{d}{dt}\, y(t) = v(t),\ \frac{d^2}{dt^2}\, y(t) = ay(t) \right\} \quad \textbf{(6)}$$

```
> subs( SUBS, RESLAMBDA ) ;
```

$$\left\{ \lambda(t) = \frac{1}{4}\, \frac{1}{x(t)^2 + y(t)^2} \left(mass \left(x(t)^2\, \omega^2 + 4\, x(t)\, u(t)\, \omega\, \zeta + y(t)^2\, \omega^2 + 4\, y(t)\, v(t)\, \omega\, \zeta - 2\, y(t)\, g + 2\, u(t)^2 + 2\, v(t)^2 - \omega^2 \right) \right) \right\} \quad \textbf{(7)}$$

Advancing with Taylor

```
> XKP1 := x(t)+u(t)*DT+ax(t)*DT^2/2 ;
  YKP1 := y(t)+v(t)*DT+ay(t)*DT^2/2 ;
  UKP1 := u(t)+ax(t)*DT ;
  VKP1 := v(t)+ay(t)*DT ;
```

$$XKP1 := x(t) + u(t)\, DT + \frac{1}{2}\, ax(t)\, DT^2$$

$$YKP1 := y(t) + v(t)\, DT + \frac{1}{2}\, ay(t)\, DT^2$$

$$UKP1 := u(t) + ax(t)\, DT$$

$$VKP1 := v(t) + ay(t)\, DT \quad \textbf{(8)}$$

Substituting acceleration

```
> XKP1 := subs( subs(SUBS,RESACC), XKP1) ;
  YKP1 := subs( subs(SUBS,RESACC), YKP1) ;
  UKP1 := subs( subs(SUBS,RESACC), UKP1) ;
  VKP1 := subs( subs(SUBS,RESACC), VKP1) ;
```

$$XKP1 := x(t) + u(t)\, DT - \frac{x(t)\, \lambda(t)\, DT^2}{mass}$$

$$YKP1 := y(t) + v(t)\, DT - \frac{1}{2}\, \frac{\left(2\, y(t)\, \lambda(t) + mass\, g\right) DT^2}{mass}$$

$$UKP1 := u(t) - \frac{2\, x(t)\, \lambda(t)\, DT}{mass}$$

$$VKP1 := v(t) - \frac{\left(2\, y(t)\, \lambda(t) + mass\, g\right) DT}{mass} \quad \textbf{(9)}$$

Substituting Lambda

```
> XKP1 := subs( subs(SUBS,RESLAMBDA), XKP1) ;
  YKP1 := subs( subs(SUBS,RESLAMBDA), YKP1) ;
  UKP1 := subs( subs(SUBS,RESLAMBDA), UKP1) ;
  VKP1 := subs( subs(SUBS,RESLAMBDA), VKP1) ;
```

$$XKP1 := x(t) + u(t)\, DT - \frac{1}{4}\, \frac{1}{x(t)^2 + y(t)^2} \left(x(t) \left(x(t)^2\, \omega^2 + 4\, x(t)\, u(t)\, \omega\, \zeta + y(t)^2\, \omega^2 \right.\right.$$

$$+ 4\, y(t)\, v(t)\, \omega\, \zeta - 2\, y(t)\, g + 2\, u(t)^2 + 2\, v(t)^2 - \omega^2 \big)\, DT^2 \big)$$

$$YKP1 := y(t) + v(t)\, DT - \frac{1}{2}\, \frac{1}{mass} \left(\left(\frac{1}{2}\, \frac{1}{x(t)^2 + y(t)^2} \big(y(t)\, mass\, \big(x(t)^2\, \omega^2 + 4\, x(t)\, u(t)\, \omega\, \zeta + y(t)^2\, \omega^2 + 4\, y(t)\, v(t)\, \omega\, \zeta - 2\, y(t)\, g + 2\, u(t)^2 + 2\, v(t)^2 - \omega^2 \big) \big) + mass\, g \right) DT^2 \right)$$

$$UKP1 := u(t) - \frac{1}{2}\, \frac{1}{x(t)^2 + y(t)^2} \big(x(t)\, \big(x(t)^2\, \omega^2 + 4\, x(t)\, u(t)\, \omega\, \zeta + y(t)^2\, \omega^2 + 4\, y(t)\, v(t)\, \omega\, \zeta - 2\, y(t)\, g + 2\, u(t)^2 + 2\, v(t)^2 - \omega^2 \big)\, DT \big)$$

$$VKP1 := v(t) - \frac{1}{mass} \left(\left(\frac{1}{2}\, \frac{1}{x(t)^2 + y(t)^2} \big(y(t)\, mass\, \big(x(t)^2\, \omega^2 + 4\, x(t)\, u(t)\, \omega\, \zeta + y(t)^2\, \omega^2 + 4\, y(t)\, v(t)\, \omega\, \zeta - 2\, y(t)\, g + 2\, u(t)^2 + 2\, v(t)^2 - \omega^2 \big) \big) + mass\, g \right) DT \right) \quad \textbf{(10)}$$

Build numerical scheme

```
> SUBSV := { x(t)=xO, y(t)=yO, u(t)=uO, v(t)=vO, mu(t)=muN } ;
```

$$SUBSV := \{ \mu(t) = muN, u(t) = uO, v(t) = vO, x(t) = xO, y(t) = yO \} \quad \textbf{(11)}$$

```
> XKP1 := subs(SUBSV,XKP1) ;
  YKP1 := subs(SUBSV,YKP1) ;
  UKP1 := subs(SUBSV,UKP1) ;
  VKP1 := subs(SUBSV,VKP1) ;
```

$$XKP1 := xO + uO\, DT - \frac{1}{4}\, \frac{1}{xO^2 + yO^2} \big(xO\, \big(\omega^2\, xO^2 + \omega^2\, yO^2 + 4\, \omega\, uO\, xO\, \zeta + 4\, \omega\, vO\, yO\, \zeta - 2\, g\, yO - \omega^2 + 2\, uO^2 + 2\, vO^2 \big)\, DT^2 \big)$$

$$YKP1 := yO + vO\, DT - \frac{1}{2}\, \frac{1}{mass} \left(\left(\frac{1}{2}\, \frac{1}{xO^2 + yO^2} \big(yO\, mass\, \big(\omega^2\, xO^2 + \omega^2\, yO^2 + 4\, \omega\, uO\, xO\, \zeta + 4\, \omega\, vO\, yO\, \zeta - 2\, g\, yO - \omega^2 + 2\, uO^2 + 2\, vO^2 \big) \big) + mass\, g \right) DT^2 \right)$$

$$UKP1 := uO - \frac{1}{2}\, \frac{1}{xO^2 + yO^2} \big(xO\, \big(\omega^2\, xO^2 + \omega^2\, yO^2 + 4\, \omega\, uO\, xO\, \zeta + 4\, \omega\, vO\, yO\, \zeta - 2\, g\, yO - \omega^2 + 2\, uO^2 + 2\, vO^2 \big)\, DT \big)$$

$$VKP1 := vO - \frac{1}{mass} \left(\left(\frac{1}{2}\, \frac{1}{xO^2 + yO^2} \big(yO\, mass\, \big(\omega^2\, xO^2 + \omega^2\, yO^2 + 4\, \omega\, uO\, xO\, \zeta + 4\, \omega\, vO\, yO\, \zeta - 2\, g\, yO - \omega^2 + 2\, uO^2 + 2\, vO^2 \big) \big) + mass\, g \right) DT \right) \quad \textbf{(12)}$$

```
> advance := proc ( x0, y0, u0, v0, dt, N )
    local kk, SUBS, x1, y1, u1, v1, XY,UV ;
    XY := [[x0,y0]] ;
```

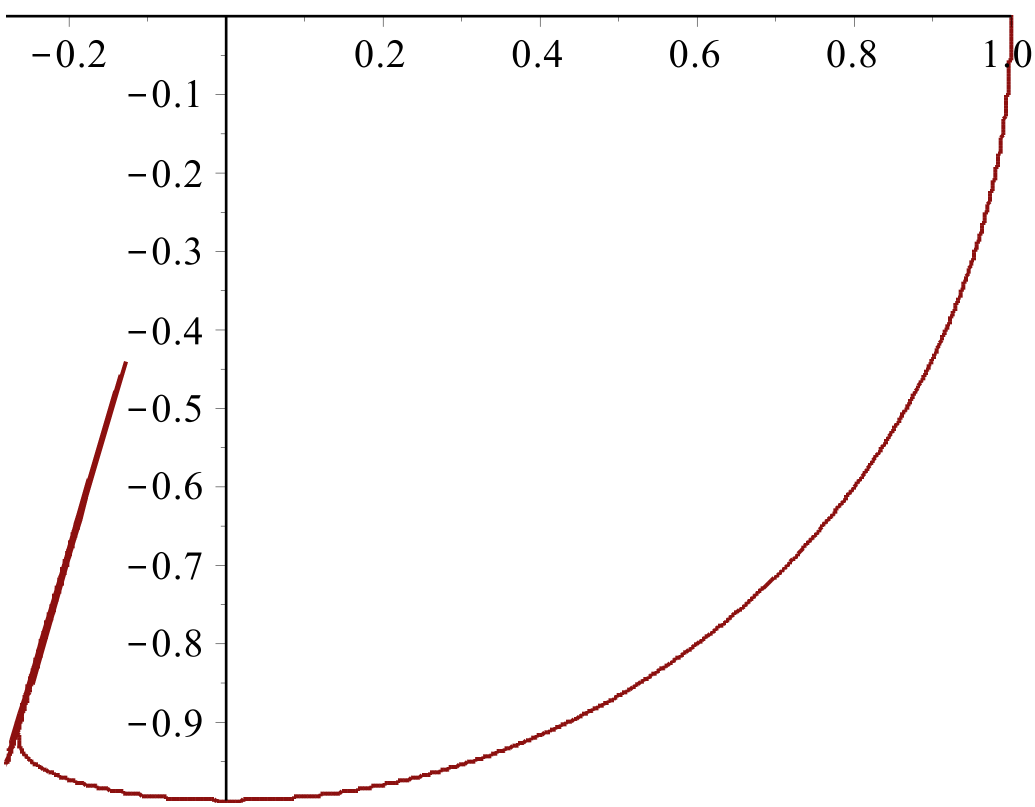
```
    UV := [[u0,v0]] ;
    for kk from 1 to N do
      SUBS := { g=9.81, DT=dt, omega=4012, zeta=1,
                xO=XY[-1][1], yO=XY[-1][2],
                uO=UV[-1][1], vO=UV[-1][2]} ;
      x1  := evalf(subs( SUBS, XKP1 )) ;
      y1  := evalf(subs( SUBS, YKP1 )) ;
      u1  := evalf(subs( SUBS, UKP1 )) ;
      v1  := evalf(subs( SUBS, VKP1 )) ;
      XY  := [op(XY),[x1,y1]] ;
      UV  := [op(UV),[u1,v1]] ;
    end  ;
    [XY,UV] ;
  end proc:
```

Test numerical scheme DT = 1/200

```
> NSTEP := 4000 ;
```

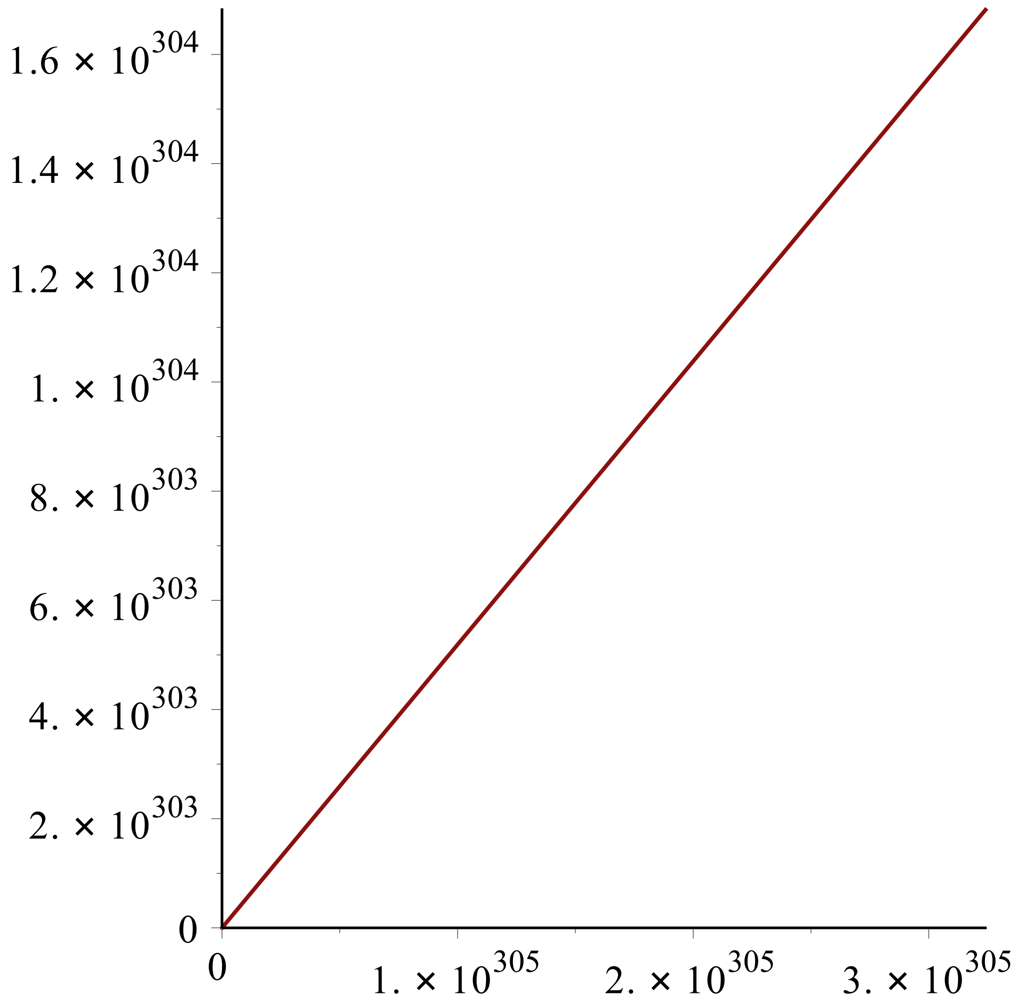
$$NSTEP := 4000 \tag{13}$$

```
> RES := advance( 1, 0, 0, 0, 1/NSTEP, NSTEP ) :
> plot( RES[1], scaling=CONSTRAINED ) ;
```



Test numerical scheme DT = 1/20

```
> RES := advance( 1, 0, 0, 0, 10/200, 150 ) :
  plot( RES[1] ) ;
```


1.6 × 10^304
1.4 × 10^304
1.2 × 10^304
1. × 10^304
8. × 10^303
6. × 10^303
4. × 10^303
2. × 10^303
0
0
1. × 10^305
2. × 10^305
3. × 10^305