Corse of Numerical Optimization

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Competition Problems

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A. Rules

The competitors must produce a MATLAB function with his/her name with accept a function, initial point (column vector), max iterates and tolerance. The function must return ha history of the iterates as a matrix whose columns are the iteration points. For example a competitor named Bartolomeo Pestalozzi must produce a MATLAB code like

```
function ITER = BartolomeoPestalozzi( FUN, X0, maxiter, tol )
.....
end
```

The dimension of the problem is deduced form `X0` or by calling `feval(FUN,'n')`. I have not produced MATLAB functions for each test case. Each competitor must write his functions, however collaboration between competitors to write the code is strongly suggested. Each function must have the following structure:

```
function RES = test1( what, x )
  global Nf_eval Ng_eval Nh_eval ;

  switch what
  case {'name','title'} ; RES = 'name_of_the_test' ;
  case {'n','size'}     ; RES = dimension of the x ;
  case 'm'              ; RES = dimension of f ;
  case {'init','start'} ; RES = init point as column vector ;
  case 'solution'       ; RES = eventual soluzion if known ;
  case 'f'  ; Nf_eval = Nf_eval+1 ;
    RES = evaluate function f(x) or the sum of the squares if multiple components ;
  case 'Df' ; Ng_eval = Ng_eval+1 ;
    RES = evaluate the gradient of f(x) ;
  case 'F'  ; Ng_eval = Ng_eval+1 ;
    RES = evaluate the non linear system F(x) ;
  case {'JF', 'DDf'} ; Nh_eval = Nh_eval+1;
    RES = evaluate the jacobian of F(x) or the Hessian of f(x) ;
  otherwise ; error( some error message ) ;
  end ;
end
```

As reference values `maxiter` is 1000 and `tol` is 10^{-8}. As stopping criteria we use:

- The infinity norm of the residual for non linear system, i.e. $\max|f_k(x_1,\ldots,x_n)|$

- The infinity norm of the gradient for minimization
- The infinity norm of $J(x_1, \dots, x_n)^T F(x_1, \dots, x_n)$ for least squares problems.

B. Nonlinear system

1. Modified Rosenbrock

The equations:

$$\begin{aligned} f(x, y) &= \frac{1}{1 + \exp(-x)} - 0.73, \\ g(x, y) &= 10(y - x^2). \end{aligned}$$

Initial Point:

$$x_0 = -1.8, \qquad y_0 = -1$$

Exact solution

$$x = \ln\left(\frac{73}{27}\right), \qquad y = \left[\ln\left(\frac{73}{27}\right)\right]^2$$

2. Powell badly Scaled

The equations:

$$\begin{aligned} f(x, y) &= 10^4 xy - 1 \\ g(x, y) &= \exp(-x) + \exp(-y) - 1.0001 \end{aligned}$$

Initial Point:

$$x_0 = 0, \qquad y_0 = 100$$

Exact solution(s)

$$\begin{aligned} x &= 4\ln(10), \qquad & y &= 0 \\ x &= 0, \qquad & y &= 4\ln(10) \end{aligned}$$

3. Freudenstein and Roth

The equations:

$$\begin{aligned} f(x, y) &= x - y(2 - y(5 - y)) - 13 \\ g(x, y) &= x - y(14 - y(1 + y)) - 29 \end{aligned}$$

Initial Point:

$$x_0 = 0.5, \qquad y_0 = -2$$

Exact solution

$$x = 5, \qquad y = 4$$

4. Semiconductor

The equation:

$$\begin{aligned} f(x,y) &= \exp(x^2+y^2)-3 \\ g(x,y) &= x+y-\sin(3(x+y)) \end{aligned}$$

Initial Point:

$$x_0 = 0.81, \qquad y_0 = 0.82$$

Exact solution

$$x = -1.01624596361444, \qquad y = 0.256625076922493$$

5. Logarithmic function (n=2)

The equations ($n = 2$):

$$f_k(x_1,\ldots,x_n) = \ln(x_k+1) - \frac{x_k}{n}, \qquad k = 1,2,\ldots,n$$

Initial Point:

$$x_1^0 = 1, \quad x_2^0 = 1, \quad \ldots \quad x_n^0 = 1$$

Exact solution(s)

$$\begin{aligned} &x_1 = 0, && x_2 = 0 \\ &x_1 = 0, && x_2 = 2.51286241725234 \\ &x_1 = 2.51286241725234, && x_2 = 0 \\ &x_1 = 2.51286241725234, && x_2 = 2.51286241725234 \end{aligned}$$

6. Logarithmic function (n=5)

The previous problem with $n = 5$. Exact solutions are $2^5 = 32$, any combinations of the two values:

$$x_k = \begin{cases} 0 \\ 13.3019952923184 \end{cases}$$

7. Discrete Integral ($n = 2$)

The equations ($n = 2$).

$$f_k(x_1,x_2,\ldots,x_n) = x_k + \frac{h}{2}\sum_{j=1}^{k} q_{kj}(x_j+t_j+1)^3, \qquad k = 1,2,\ldots,n$$

where

$$h = \frac{1}{n+1}, \qquad t_k = h\,k, \qquad q_{kj} = \begin{cases} (1-t_k)t_j & \text{if } j \le k \\ (1-t_j)t_k & \text{if } j > k \end{cases}$$

initial point

$$x_k^0 = t_k(t_k-1), \qquad k = 1,2,\ldots,n$$

Exact solution

$$x_1 = -0.0739748874643476, \qquad x_2 = -0.162925156390870$$

8. Discrete Integral (n=5)

The previous problem with $n = 5$.
Exact solution

$$\begin{aligned} x_1 &= -1.75616539435172, \\ x_2 &= -5.29228466926422, \\ x_3 &= -9.72022156411245, \\ x_4 &= -0.13176631159597, \\ x_5 &= -0.123717020772933 \end{aligned}$$

9. Discrete Boundary Value

The equations ($n = 2$).

$$f_k(x_1, x_2, \ldots, x_n) = 2x_k - x_{k-1} - x_{k+1} + \frac{h^2}{2}(x_k + t_k + 1)^3, \qquad k = 1, 2, \ldots, n$$

where

$$h = \frac{1}{n+1}, \qquad t_k = h\,k, \qquad x_0 = x_{n+1} = 0$$

initial point

$$x_k^0 = t_k(t_k - 1) \qquad k = 1, 2, \ldots, n$$

Exact solution

$$x_1 = -0.128246763033732, \qquad x_2 = -0.159267567244641$$

10. Discrete Boundary Value (n=5)

The previous problem with $n = 5$.
Exact solution

$$\begin{aligned} x_1 &= -0.0750221292923205, \\ x_2 &= -0.131976210352191, \\ x_3 &= -0.164848771909337, \\ x_4 &= -0.164664680215801, \\ x_5 &= -0.117417651684194 \end{aligned}$$

C. Least squares problems

1. Penalty function I ($n = 2$)

Minimize $f_1^2 + f_2^2 + \cdots + f_{n+1}^2$ where $n = 2$ and

$$f_k(x_1, x_2, \ldots, x_n) = \frac{x_k - 1}{\sqrt{100000}}, \qquad k = 1, 2, \ldots, n$$

$$f_{n+1}(x_1, x_2, \ldots, x_n) = x_1^2 + x_2^2 + \cdots + x_n^2 - \frac{1}{4}$$

Initial Point:

$$x_k^0 = k, \qquad k = 1, 2, \ldots, n$$

Exact solution

$$x_1 = x_2 = 0.353559854817441$$

2. Penalty function I (n=5)

The previous problem with $n = 5$.
Exact solution

$$x_1 = x_2 = x_3 = x_4 = x_5 = 0.223614561200051$$

3. Penalty function I (n=11)

The previous problem with $n = 11$.

$$x_1 = \cdots = x_{11} = 0.150764163929764$$

4. Penalty function II ($n = 2$)

Minimize $f_1^2 + f_2^2 + \cdots + f_{2n}^2$ where $n = 2$ and

$$f_1(x_1, \ldots, x_n) = x_1 - 0.2,$$

$$f_k(x_1, \ldots, x_n) = a\left(\exp\left(b\, x_k\right) + \exp\left(b\, x_{k-1}\right) - y_k\right), \qquad k = 2, 3, \ldots, n$$

$$f_k(x_1, \ldots, x_n) = a\left(\exp\left(b\, x_{k-n+1}\right) + \exp\left(-b\right)\right), \qquad k = n+1, n+2, \ldots, 2n-1$$

$$f_{2n}(x_1, \ldots, x_n) = \left(\sum_{j=1}^{n} (n - j + 1) x_j^2\right) - 1,$$

where

$$a = \sqrt{10^{-5}}, \qquad b = \frac{1}{10}, \qquad y_k = \exp\left(b\, k\right) + \exp\left(b\,(k-1)\right),$$

Initial Point:

$$x_k^0 = \frac{1}{2}, \qquad k = 1, 2, \ldots, n$$

5. Penalty function II (n=5)

The previous problem with $n = 5$.

6. Penalty function II (n=11)

The previous problem with $n = 11$.

7. Jennrigh and Sampson

Minimize $f_1^2 + f_2^2 + f_3^2 + f_4^2 + f_5^2$ where

$$f_k(x, y) = 2 + 2k - (\exp(x) + \exp(y)), \qquad k = 1, 2, \ldots, 5$$

Initial Point:

$$x_0 = 0.3, \qquad y_0 = 0.4$$

Exact solution

$$x = 2.079441385, \qquad y = -13.58886304$$

8. Meyer function

Minimize $f_1^2 + f_2^2 + \cdots + f_{16}^2$ where

$$\begin{aligned} f_k(x, y, z) &= x \exp\left(\frac{y}{z + 45 + 5k}\right) - w_k, \qquad k = 1, 2, \ldots, 16 \\ \boldsymbol{w} &= (34780,\ 28610,\ 23650,\ 19630,\ 16370,\ 13720,\ 11540, \\ &\quad 9744,\ 8261,\ 7030,\ 6005,\ 5147,\ 4427,\ 3820,\ 3307,\ 2872) \end{aligned}$$

Initial Point:

$$x_0 = 0.02, \qquad y_0 = 4000, \qquad z_0 = 250$$

9. Box $3D$

Minimize $f_1^2 + f_2^2 + f_3^2 + f_4^2$ where

$$f_k(x, y, z) = \exp(-b\,k\,x) - \exp(-b\,k\,y) - z\left(\exp(-b\,k) - \exp(-k)\right), \qquad k = 1, 2, 3, 4.$$

where $b = 1/10$ and initial point:

$$x_0 = 0, \qquad y_0 = 10, \qquad z_0 = 20$$

Exact solution

$$x = 1, \qquad y = -10, \qquad z = 1$$

10. Minimal function

Minimize $f_1^2 + f_2^2 + \cdots + f_n^2$ where $n = 5$ and

$$f_k(x_1, x_2, \ldots, x_n) = \ln(x_k) + \exp(x_k) - \sqrt{(\ln(x_k) - \exp(x_k))^2 + 10^{-4}}, \qquad k = 1, 2, \ldots, n$$

Initial Point:

$$x_k^0 = 1, \qquad k = 1, 2, \ldots, n$$

Exact solution

$$x_k = 1.000009197 \qquad k = 1, 2, \ldots, n$$

D. Minimization

1. Wood

Minimize

$$\begin{aligned} f(x, y, z, w) = &\ 100(y - x^2)^2 + (x - 1)^2 + 90(w - z^2)^2 + (1 - z)^2 \\ &+10.1[(y - 1)^2 + (w - 1)^2] + 19.8(y - 1)(w - 1) \end{aligned}$$

Initial Point:

$$x = y = z = w = 0$$

Exact solution

$$x = y = z = w = 1,$$

2. Easom

Minimize

$$f(x, y) = -\cos(x)\cos(y)\exp(-(x - \pi)^2 - (y - \pi)^2)$$

Initial Point:

$$x = y = 50$$

Solutions

$$x = 3.14159265358979, \qquad y = 3.14159265358979$$
$$x = 1.30499545043767, \qquad y = 1.30499545043767$$
$$x = 4.97818985674192, \qquad y = 4.97818985674192$$

3. Goldstein-Price

Minimize

$$\begin{aligned} f(x, y) = &\left[1 + (x + y + 1)^2(19 - 14x + 3x^2 - 14y + 6xy + 3y^2\right] \\ &\left[30 + (2x - 3y)^2(18 - 32x + 12x^2 + 48y - 36xy + 27y^2)\right]. \end{aligned}$$

Initial Point:

$$x_0 = 0, \qquad y_0 = -1$$

Solutions

$$x = -\frac{3}{5}, \qquad y = -\frac{2}{5}$$
$$x = -\frac{6}{5}, \qquad y = -\frac{4}{5}$$
$$x = 0, \qquad y = -1$$

4. Six-hump camel back

Minimize

$$f(x, y) = \left(4 - 2.1x^2 + \frac{x^4}{3}\right) x^2 + xy + 4(y^2 - 1)y^2$$

Initial Point:

$$x_0 = 1, \qquad y_0 = 1$$

Solutions

$$x = +0.08984201310, \qquad y = -0.7126564030$$
$$x = -0.08984201310, \qquad y = +0.7126564030$$
$$x = +1.607104753, \qquad y = +0.5686514549$$
$$x = -1.607104753, \qquad y = -0.5686514549$$
$$x = +1.703606715, \qquad y = -0.7960835687$$
$$x = -1.703606715, \qquad y = +0.7960835687$$

5. Problem C.7.

6. Problem C.8.

7. Problem C.9.

8. Problem C.10. with $n = 2$

9. Problem C.10. with $n = 7$

10. Problem C.10. with $n = 11$