Interpolazione Polinomiale: Fenomeno di Runge

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> # definisce la funzione da interpolare
  fun := x -> 1/(1+x^2) ;
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$$fun := x \rightarrow \frac{1}{1 + x^2}$$

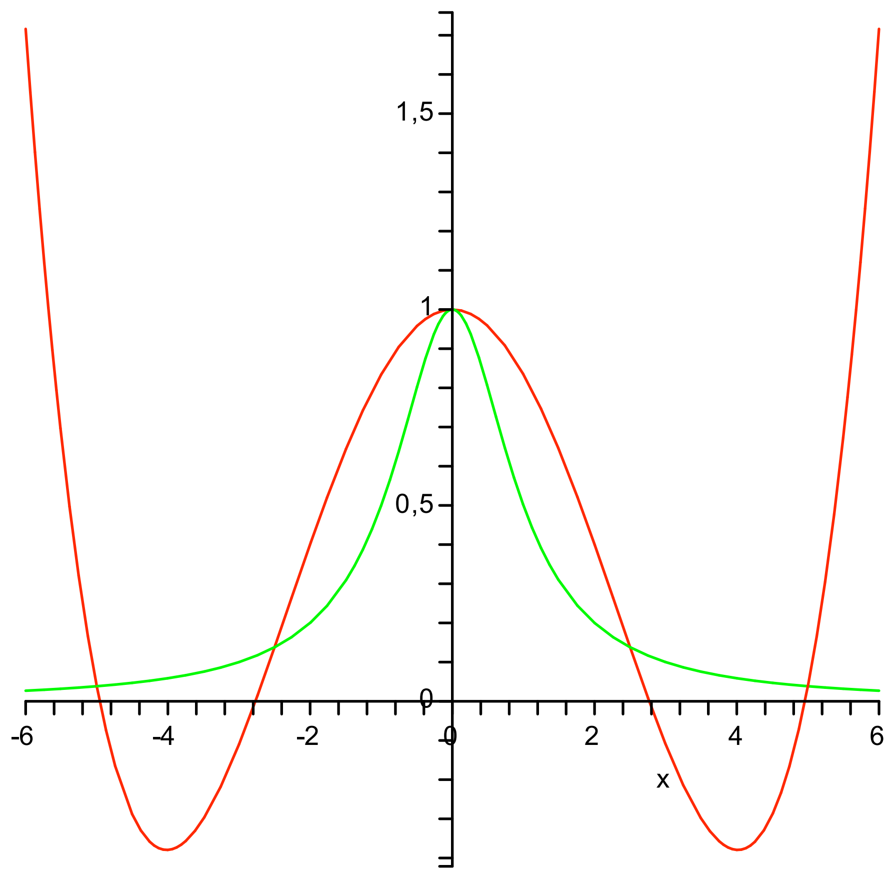
```
> # funzione che costruisce i funti di interpolazione
  pnts := proc (a,b,n,fun)
    local X, Y, i, k, mappa ;
    # definisce la funzione che mappa [1,n] in [a,b]
    mappa := (n,x) -> (b-a) * (x-1) / (n-1) + a ;
    X := [seq(mappa(n,k),k=1..n)] ;
    Y := [seq(fun(X[i]),i=1..nops(X))] ;
    return X, Y ;
  end proc ;
```

```
pnts := proc(a, b, n, fun)
local X, Y, i, k, mappa;
    mappa := proc(n, x) option operator, arrow; ((b - a)*(x - 1))/(n - 1) + a end proc;;
    X := [seq(mappa(n, k), k = 1 .. n)];
    Y := [seq(fun(X[i]), i = 1 .. nops(X))];
    return X, Y;
end proc;
```

```
> X5, Y5 := pnts(-5,5,5,fun) ;
  p5     := interp(X5,Y5,x) ;
  plot([p5,fun(x)],x=-6..6) ;
```

$$X5, Y5 := \left[-5, \frac{-5}{2}, 0, \frac{5}{2}, 5\right], \left[\frac{1}{26}, \frac{4}{29}, 1, \frac{4}{29}, \frac{1}{26}\right]$$

$$p5 := \frac{2}{377} x^4 - \frac{129}{754} x^2 + 1$$



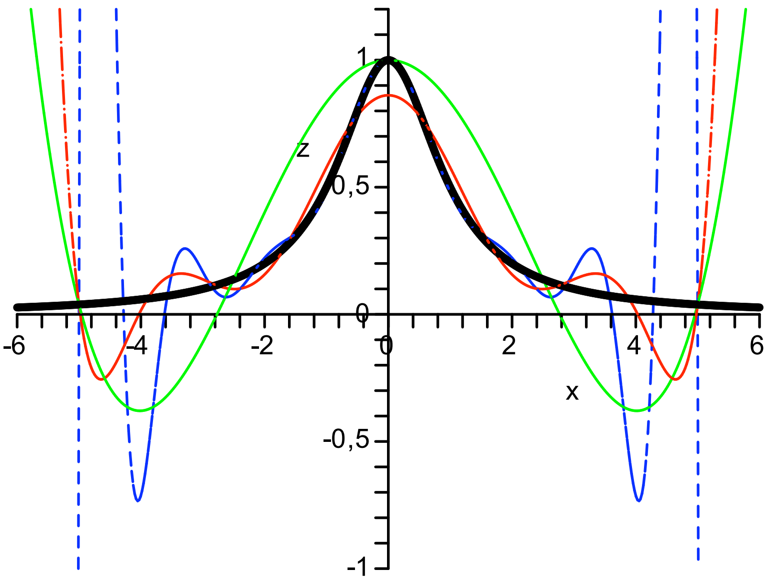

```
> X10, Y10 := pnts(-5,5,10,fun) :
  p10      := interp(X10,Y10,x) ;
```

$$p10 := \frac{4782969}{86398461344} x^8 - \frac{124180047}{43199230672} x^6 + \frac{530979543}{10799807668} x^4 - \frac{14274621297}{43199230672} x^2 + \frac{74435570719}{86398461344}$$

```
> X15, Y15 := pnts(-5,5,15,fun) :
  p15      := interp(X15,Y15,x) ;
```

$$p15 := -\frac{1212165753228161}{1409919659478161} x^2 + \frac{48168981468272}{108455358421397} x^4 + 1 - \frac{25979418014669}{216910716842794} x^6 + \frac{790304686871}{46899614452496} x^8 - \frac{2145238101727}{1735285734742352} x^{10} + \frac{5931980229}{133483518057104} x^{12} - \frac{13841287201}{22558714551650576} x^{14}$$

```
> plot([fun(x),p5,p10,p15],x=-6..6,z=-1..1.2,
      axes=NORMAL,
      numpoints=500, color=[black,green,red,blue],
      linestyle=[1,1,2,3],
      thickness=[3,1,1,1]) ;
```



z
1
0,5
0
-0,5
-1
-6
-4
-2
0
2
4
6
x

>